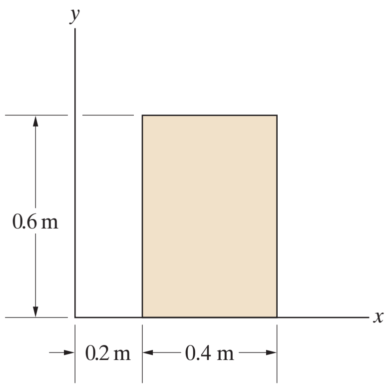
Problem 8.1 Use the method described in Active Example 8.1 to determine I_Y and k_y for the rectangular area.



Solution: The height of the vertical strip of width dx is 0.6 m, so the area is $dA = (0.6\text{ m})\,dx$.

We can use this expression to determine I_y.

$$I_y = \int_A x^2\,dx = (0.6\text{ m})\int_{0.2\text{m}}^{0.4\text{ m}} x^2\,dx$$

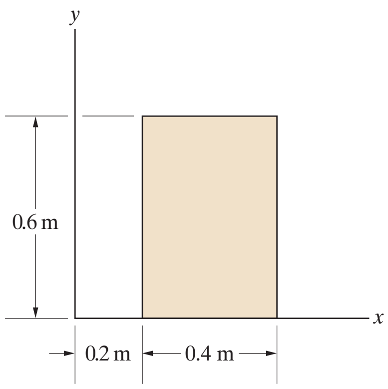
$$= (0.6\text{ m})\left[\frac{x^3}{3}\right]_{0.2\text{ m}}^{0.4\text{ m}} = 0.0416\text{ m}^4$$

The radius of gyration about the y axis is

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.0416\text{ m}^4}{(0.4\text{ m})\,(0.6\text{ m})}} = 0.416\text{ m}$$

$$\boxed{I_y = 0.0416\text{ m}^4,\ k_y = 0.416\text{ m}}$$

Problem 8.2 Use the method described in Active Example 8.1 to determine I_x and k_x for the rectangular area.



Solution: It was shown in Active Example 8.1 that the moment of inertia about the x axis of a vertical strip of width dx and height $f(x)$ is

$$(I_x)_{\text{strip}} = \frac{1}{3}\,[f(x)]^3\,dx.$$

For the rectangular strip, $f(x) = 0.6$ m. Integrating to determine I_x for the rectangular area.,

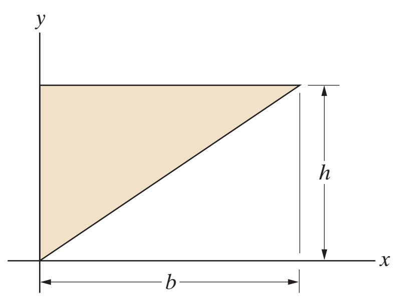
$$I_x = \int_{0.2\text{m}}^{0.4\text{m}} \frac{1}{3}\,(0.6\text{ m})^3\,dx = \frac{1}{3}\,(0.6\text{ m})^3\,[x]_{0.2\text{m}}^{0.4\text{m}} = 0.0288\text{ m}^4$$

The radius of gyration about the x axis is

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.0288\text{ m}^4}{(0.4\text{ m})(0.6\text{ m})}} = 0.346\text{ m}$$

$$\boxed{I_x = 0.0288\text{ m}^4,\ k_y = 0.346\text{ m}}$$

Problem 8.3 In Active Example 8.1, suppose that the triangular area is reoriented as shown. Use integration to determine I_y and k_y.



Solution: The height of a vertical strip of width dx is $h - (h/b)x$, so the area

$$dA = \left(h - \frac{h}{b}x\right)\,dx.$$

We can use this expression to determine I_y:

$$I_y = \int_A x^2\,dA = \int_0^b x^2\left(h - \frac{h}{b}x\right)\,dx = h\left[\frac{x^3}{3} - \frac{x^4}{4b}\right]_0^b = \frac{1}{12}hb^3$$

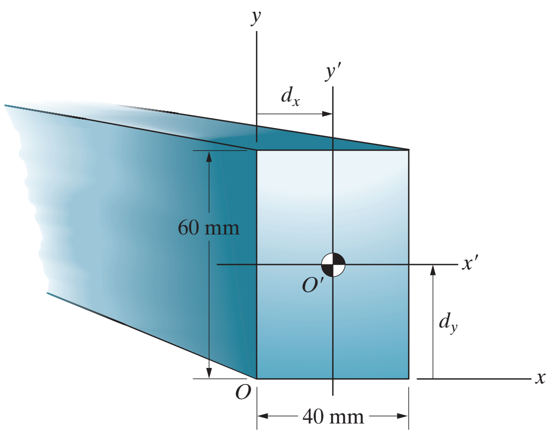
The radius of gyration about the y axis is

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{1}{12}hb^3}{\frac{1}{2}hb}} = \frac{b}{\sqrt{6}}$$

$$\boxed{I_y = \frac{hb^3}{12},\ k_y = \frac{b}{\sqrt{6}}.}$$

Problem 8.4 (a) Determine the moment of inertia I_y of the beam's rectangular cross section about the y axis.

(b) Determine the moment of inertia $I_{y'}$ of the beam's cross section about the y' axis. Using your numerical values, show that $I_y = I_{y'} + d_x^2 A$, where A is the area of the cross section.



Solution:

(a) $$\boxed{I_y = \int_0^{40\text{ mm}} \int_0^{60\text{ mm}} x^2\,dy\,dx = 1.28 \times 10^6 \text{ mm}^4}$$

(b) $$\boxed{I_{y'} = \int_{-20\text{ mm}}^{20\text{ mm}} \int_{-30\text{ mm}}^{30\text{ mm}} x^2\,dy\,dx = 3.2 \times 10^5 \text{ mm}^4}$$

$$I_y = I_{y'} + d_x{}^2 A$$

$$1.28 \times 10^6 \text{ mm}^4 = 3.2 \times 10^5 \text{ mm}^4 + (20\text{ mm})^2[(40\text{ mm})(60\text{ mm})]$$

Problem 8.5 (a) Determine the polar moment of inertia J_O of the beam's rectangular cross section about the origin O.

(b) Determine the polar moment of inertia $J_{O'}$ of the beam's cross section about the origin O'. Using your numerical values, show that $J_O = J_{O'} + (d_x^2 + d_y^2)A$, where A is the area of the cross section.

Solution:

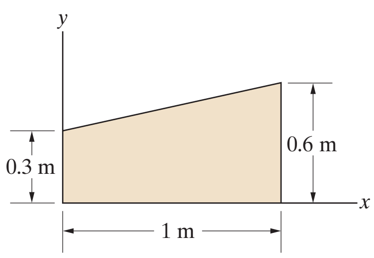
(a) $$\boxed{J_O = \int_0^{40\text{ mm}} \int_0^{60\text{ mm}} (x^2 + y^2)\,dy\,dx = 4.16 \times 10^6 \text{ mm}^4}$$

(b) $$\boxed{J_{O'} = \int_{-20\text{ mm}}^{20\text{ mm}} \int_{-30\text{ mm}}^{30\text{ mm}} (x^2 + y^2)\,dy\,dx = 1.04 \times 10^6 \text{ mm}^4}$$

(c) $$J_O = J_{O'} + (d_x{}^2 + d_y{}^2)A$$

$$4.16 \times 10^6 \text{ mm}^4 = 1.04 \times 10^6 \text{ mm}^4 + [(20\text{ mm})^2 + (30\text{ mm})^2][(40\text{ mm})(60\text{ mm})]$$

Problem 8.6 Determine I_y and k_y.



Solution:

$$A = (0.3\text{ m})(1\text{ m}) + \frac{1}{2}(0.3\text{ m})(1\text{ m}) = 0.45\text{ m}^2$$

$$\boxed{I_y = \int_0^{1\text{ m}} \int_0^{0.3\text{ m}+0.3x} x^2\,dy\,dx = 0.175\text{ m}^4}$$

$$\boxed{k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.175\text{ m}^4}{0.45\text{ m}^2}} = 0.624\text{ m}}$$

Problem 8.7 Determine J_O and k_O.

Solution:

$$A = (0.3 \text{ m})(1 \text{ m}) + \frac{1}{2}(0.3 \text{ m})(1 \text{ m}) = 0.45 \text{ m}^2$$

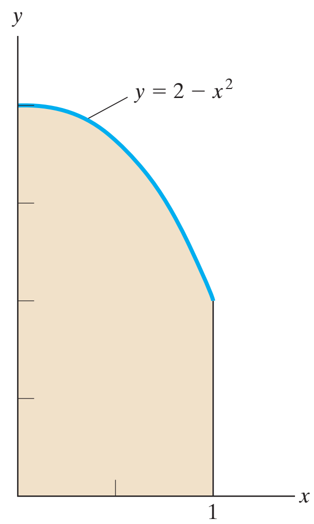
$$\boxed{\begin{aligned} J_O &= \int_0^{1\,\text{m}} \int_0^{0.3\,\text{m}+0.3x} (x^2 + y^2)\,dy\,dx = 0.209 \text{ m}^4 \\ k_O &= \sqrt{\frac{0.209 \text{ m}^4}{0.45 \text{ m}^2}} = 0.681 \text{ m} \end{aligned}}$$

Problem 8.8 Determine I_{xy}.

Solution:

$$\boxed{I_{xy} = \int_0^{1\,\text{m}} \int_0^{0.3\,\text{m}+0.3x} xy\,dy\,dx = 0.0638 \text{ m}^4}$$

Problem 8.9 Determine I_y.



Solution: The height of a vertical strip of width dx is $2 - x^2$, so the area is

$$dA = (2 - x^2)\,dx.$$

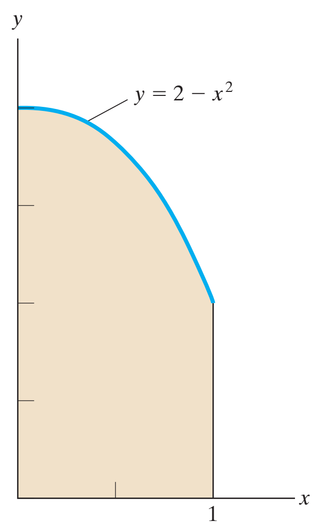
We can use this expression to determine I_y:

$$I_y = \int_A x^2\,dA = \int_0^1 x^2(2 - x^2)\,dx = \left[\frac{2x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$= 0.467.$$

$$\boxed{I_y = 0.467.}$$

Problem 8.10 Determine I_x.



Solution: It was shown in Active Example 8.1 that the moment of inertia about the x axis of a vertical strip of width dx and height $f(x)$ is

$$(I_x)_{\text{strip}} = \frac{1}{3}\,[f(x)]^3\,dx.$$

In this problem $f(x) = 2 - x^3$. Integrating to determine I_x for the area,

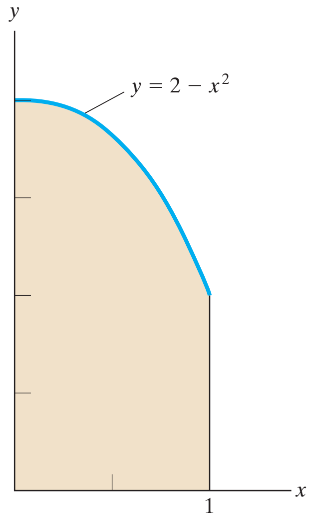
$$I_x = \int_0^1 \frac{1}{3}(2 - x^2)^3\,dx$$

$$= \frac{1}{3}\int_0^1 (8 - 12x^2 + 6x^4 - x^6)\,dx$$

$$= \frac{1}{3}\left[8x - \frac{12x^3}{3} + \frac{6x^5}{5} - \frac{x^7}{7}\right]_0^1 = 1.69.$$

$$\boxed{I_x = 1.69.}$$

Problem 8.11 Determine J_O.

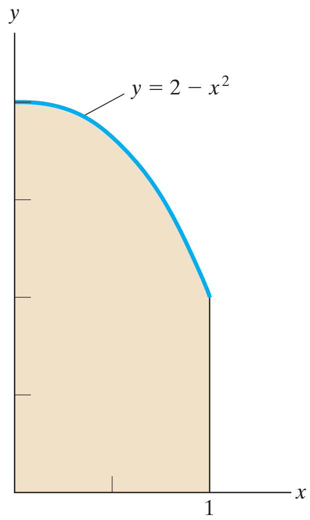


Solution: See the solutions to Problems 8.9 and 8.10. The polar moment of inertia is

$$J_O = I_x + I_y = 1.69 + 0.467 = 2.15.$$

$\boxed{J_O = 2.15.}$

Problem 8.12 Determine I_{xy}.



Solution: It was shown in Active Example 8.1 that the product of inertia of a vertical strip of width dx and height $f(x)$ is

$$(I_{xy})_{\text{strip}} = \frac{1}{2}[f(x)]^2 x dx.$$

In this problem, $f(x) = 2 - x^2$. Integrating to determine I_{xy} for the area,

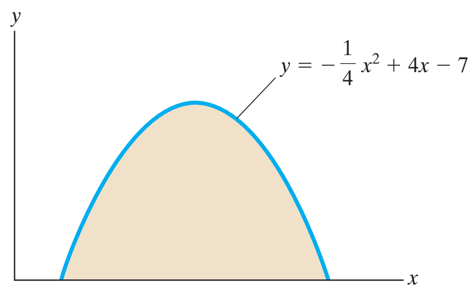
$$I_{xy} = \int_0^1 \frac{1}{2}(2 - x^2)^2 x dx$$

$$= \frac{1}{2}\int_0^1 (4x - 4x^3 + x^5)x$$

$$= \frac{1}{2}\left[\frac{4x^2}{2} - \frac{4x^4}{4} + \frac{x^6}{6}\right]_0^1 = 0.583.$$

$\boxed{I_{xy} = 0.583.}$

Problem 8.13 Determine I_y and k_y.



Solution: First we need to locate the points where the curve intersects the x axis.

$$-\frac{1}{4}x^2 + 4x - 7 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(-1/4)(-7)}}{2(-1/4)} = 2, 14$$

Now $A = \int_2^{14}\int_0^{-x^2/4+4x-7} dy dx = 72$

$$\boxed{\begin{aligned} I_y &= \int_2^{14}\int_0^{-x^2/4+4x-7} x^2 dy dx = 5126 \\ k_y &= \sqrt{\frac{I_y}{A}} = \sqrt{\frac{5126}{72}} = 8.44 \end{aligned}}$$

Problem 8.14 Determine I_x and k_x.

Solution: See Solution to Problem 8.13

$$\boxed{\begin{aligned} I_x &= \int_2^{14}\int_0^{-x^2/4+4x-7} y^2\,dy\,dx = 1333 \\ k_x &= \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1333}{72}} = 4.30 \end{aligned}}$$

Problem 8.15 Determine J_O and k_O.

Solution: See Solution to 8.13 and 8.14

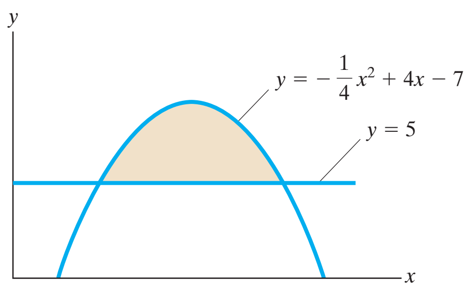
$$\boxed{\begin{aligned} J_O &= I_x + I_y = 1333 + 5126 = 6459 \\ k_O &= \sqrt{\frac{J_O}{A}} = \sqrt{\frac{6459}{72}} = 9.47 \end{aligned}}$$

Problem 8.16 Determine I_{xy}.

Solution:

$$\boxed{I_{xy} = \int_2^{14}\int_0^{-x^2/4+4x-7} xy\,dy\,dx = 2074}$$

Problem 8.17 Determine I_y and k_y.



Solution: First we need to locate the points where the curve intersects the line.

$$-\frac{1}{4}x^2 + 4x - 7 = 5 \Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(-1/4)(-12)}}{2(-1/4)} = 4, 12$$

$$A = \int_4^{12}\int_5^{-x^2/4+4x-7} dy\,dx = 21.33$$

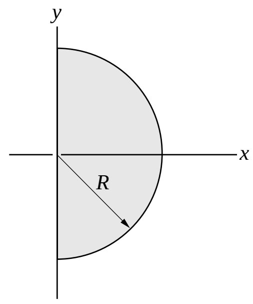
$$\boxed{\begin{aligned} I_y &= \int_4^{12}\int_5^{-x^2/4+4x-7} x^2\,dy\,dx = 1434 \\ k_y &= \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1434}{21.33}} = 8.20 \end{aligned}}$$

Problem 8.18 Determine I_x and k_x.

Solution: See Solution to Problem 8.17

$$\boxed{\begin{aligned} I_x &= \int_4^{12}\int_5^{-x^2/4+4x-7} y^2\,dy\,dx = 953 \\ k_x &= \sqrt{\frac{I_x}{A}} = \sqrt{\frac{953}{21.33}} = 6.68 \end{aligned}}$$

Problem 8.19 (a) Determine I_y and k_y by letting dA be a vertical strip of width dx.
(b) The polar moment of inertia of a circular area with its center at the origin is $J_O = \frac{1}{2}\pi R^4$. Explain how you can use this information to confirm your answer to (a).



Solution: The equation of the circle is $x^2 + y^2 = R^2$, from which $y = \pm\sqrt{R^2 - x^2}$. The strip dx wide and y long has the elemental area $dA = 2\sqrt{R^2 - x^2}\,dx$. The area of the semicircle is

$$A = \frac{\pi R^2}{2} I_y = \int_A x^2\,dA = 2\int_0^R x^2\sqrt{R^2 - x^2}\,dx$$

$$= 2\left[-\frac{x(R^2 - x^2)^{3/2}}{4} + \frac{R^2 x(R^2 - x^2)^{1/2}}{8} + \frac{R^4}{8}\sin^{-1}\left(\frac{x}{R}\right)\right]_0^R$$

$$= \frac{\pi R^4}{8}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \frac{R}{2}$$

(b) If the integration were done for a circular area with the center at the origin, the limits of integration for the variable x would be from $-R$ to R, doubling the result. Hence, doubling the answer above,

$$I_y = \frac{\pi R^4}{4}.$$

By symmetry, $I_x = I_y$, and the polar moment would be

$$J_O = 2I_y = \frac{\pi R^4}{2},$$

which is indeed the case. Also, since $k_x = k_y$ by symmetry for the full circular area,

$$k_O = \sqrt{\frac{I_x}{A} + \frac{I_y}{A}} = \sqrt{2\frac{I_y}{A}} = \sqrt{\frac{J_O}{A}}$$

as required by the definition. Thus the result checks.

Problem 8.20 (a) Determine I_x and k_x for the area in Problem 8.19 by letting dA be a horizontal strip of height dy.
(b) The polar moment of inertia of a circular area with its center at the origin is $J_O = \frac{1}{2}\pi R^4$. Explain how you can use this information to confirm your answer to (a).

Solution: Use the results of the solution to Problem 8.19, $A = \frac{\pi R^2}{2}$. The equation for the circle is $x^2 + y^2 = R^2$, from which $x = \pm\sqrt{R^2 - y^2}$. The horizontal strip is from 0 to R, hence the element of area is

$$dA = \sqrt{R^2 - y^2}\,dy.$$

$$I_x = \int_A y^2\,dA = \int_{-R}^{+R} y^2\sqrt{R^2 - y^2}\,dy$$

$$= \left[-\frac{y(R^2 - y^2)^{3/2}}{4} + \frac{R^2 y(R^2 - y^2)^{1/2}}{8} + \frac{R^4}{8}\sin^{-1}\left(\frac{y}{R}\right)\right]_{-R}^R$$

$$= \left[\frac{R^4}{8}\frac{\pi}{2} + \frac{R^4}{8}\frac{\pi}{2}\right] = \frac{\pi R^4}{8}$$

$$k_x = \sqrt{\frac{I_x}{A}} = \frac{R}{2}.$$

(b) If the area were circular, the strip would be twice as long, and the moment of inertia would be doubled:

$$I_x = \frac{\pi R^4}{4}.$$

By symmetry $I_y = I_x$,

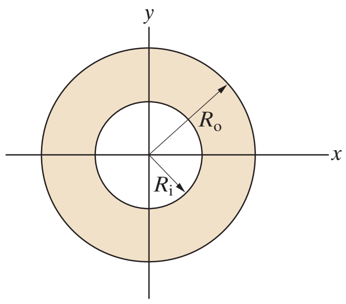
$$\text{and } J_O = 2I_x = \frac{\pi R^4}{2},$$

which is indeed the result. Since $k_x = k_y$ by symmetry for the full circular area, the

$$k_O = \sqrt{\frac{I_x}{A} + \frac{I_y}{A}} = \sqrt{2\frac{I_x}{A}} = \sqrt{\frac{J_O}{A}}$$

as required by the definition. This checks the answer.

Problem 8.21 Use the procedure described in Example 8.2 to determine the moment of inertia I_x and I_y for the annular ring.

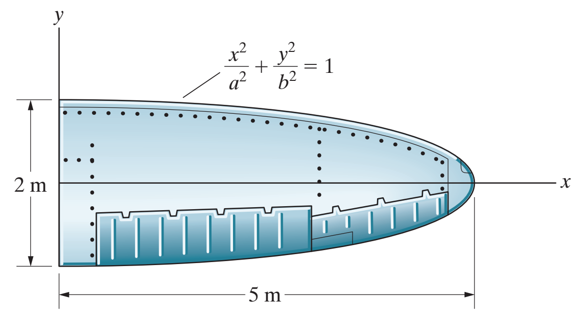


Solution: We first determine the polar moment of inertia J_O by integrating in terms of polar coordinates. Because of symmetry and the relation $J_O = I_x + I_y$, we know that I_x and I_y each equal $\frac{1}{2}J_O$. Integrating as in Example 8.2, the polar moment of inertia for the annular ring is

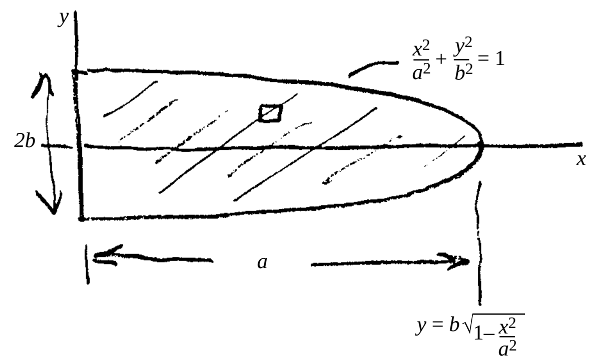
$$J_O = \int_A r^2 dA = \int_{R_i}^{R_o} r^2 (2\pi r) dr = \frac{1}{2}\pi(R_o^4 - R_i^4)$$

Therefore $\boxed{I_x = I_y = \frac{1}{4}\pi(R_o^4 - R_i^4)}$

Problem 8.22 What are the values of I_y and k_y for the elliptical area of the airplane's wing?



Solution:



$$I_y = \int_A x^2\,dA = \int_{-0}^{a}\int_{-y}^{y} x^2\,dy\,dx$$

$$I_y = 2\int_0^a\int_0^y x^2\,dy\,dx$$

$$I_y = 2\int_0^a [x^2 y]_0^{b(1-\frac{x^2}{a^2})^{1/2}}\,dx$$

$$I_y = 2\int_0^a x^2 b\left(1-\frac{x^2}{a^2}\right)^{1/2} dx$$

$$I_y = 2b\int_0^a x^2\sqrt{1-\frac{x^2}{a^2}}\,dx$$

Rewriting

$$I_y = \frac{2b}{a}\int_0^a x^2\sqrt{a^2-x^2}\,dx$$

$$I_y = \frac{2b}{a}\left[-\frac{x(a^2-x^2)^{3/2}}{4} + \frac{a^2x\sqrt{a^2-x^2}}{8} + \frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

(from the integral tables)

$$I_y = \frac{2b}{a}\left\{\left[-\cancelto{0}{\frac{a(a^2-a^2)^{3/2}}{4}} + \cancelto{0}{\frac{a^3\sqrt{a^2-a^2}}{8}} + \frac{a^4}{8}\sin^{-1}\left(\frac{a}{a}\right)\right] - \left[\cancelto{0}{\frac{0(a^2)^{3/2}}{4}} + \cancelto{0}{\frac{a^2\cdot 0\sqrt{a^2}}{8}} + \frac{a^4}{8}\cancelto{0}{\sin^{-1}}\left(\frac{0}{a}\right)\right]\right\}$$

$$I_y = \frac{\cancel{2}b}{a}\,\frac{a^4}{8}\,\frac{\pi}{\cancel{2}}$$

$$I_y = \frac{2a^3b\pi}{8}$$

Evaluating, we get

$$I_y = 49.09\text{ m}^4$$

The area of the ellipse (half ellipse) is

$$A = 2\int_0^a\int_0^{b\left(1-\frac{x^2}{a}\right)^{1/2}} dy\,dx$$

$$A = 2\int_0^a b\left(1-\frac{x^2}{a}\right) dx$$

$$= \frac{2b}{a}\int_0^a (a^2-x^2)^{1/2}\,dx$$

$$= \frac{2b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \frac{2b}{a}\left[\left(\frac{a\sqrt{0}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right) - \left(\frac{0\sqrt{a}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{0}{a}\right)\right)\right]$$

$$A = \frac{\cancel{2}b}{\cancel{a}}\,\frac{\cancel{a^2}}{\cancel{2}}\,\frac{\pi}{2} = \frac{\pi ab}{2}$$

Evaluating, we get

$$A = 7.85\text{ m}^2$$

Finally

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{49.09}{7.85}}$$

$$k_y = 2.5\text{ m}$$

Problem 8.23 What are the values of I_x and k_x for the elliptical area of the airplane's wing in Problem 8.22?

Solution:

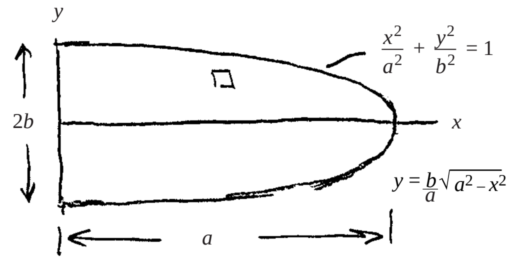
$$I_x = \int_A y^2\,dA = 2\int_0^a \int_0^{y=\frac{b}{a}\sqrt{a^2-x^2}} y^2\,dy\,dx$$

$$I_x = 2\int_0^a \left[\frac{y^3}{3}\right]_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$I_x = 2\int_0^a \frac{b^3}{3a^3}(a^2-x^2)^{3/2}\,dx$$

$$I_x = \frac{2b^3}{3a^3}\left[\frac{x(a^2-x^2)^{3/2}}{4} + \frac{3a^2x\sqrt{a^2-x^2}}{8} + \frac{3}{8}a^4\sin^{-1}\left(\frac{x}{a}\right)\right]\Bigg|_0^a$$

$$I_x = \frac{2b^3}{3a^3}\left[\frac{a(0)}{4} + \frac{3a^3\sqrt{0}}{8} + \frac{3}{8}a^4\frac{\pi}{2} - \frac{0(a^2)}{4} - \frac{3a^2\cdot 0\sqrt{a^2}}{8} + 0\right]$$



$$I_x = \frac{\cancel{2}b^3}{\cancel{3}a^{\cancel{3}}}\cdot\left(\frac{\cancel{3}}{8}\right)a^{\cancel{4}}\left(\frac{\pi}{\cancel{2}}\right)$$

$$I_x = \frac{\cancel{3}ab^3\pi}{\cancel{3}.8} = \frac{ab^3\pi}{8}$$

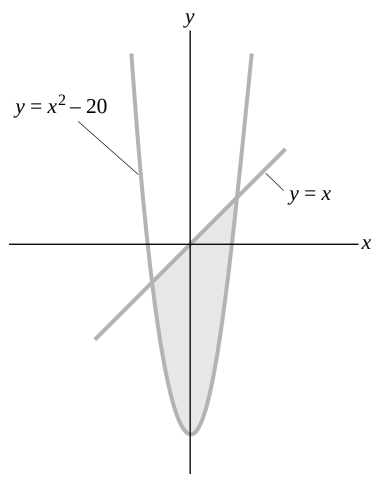
Evaluating ($a = 5$, $b = 1$)

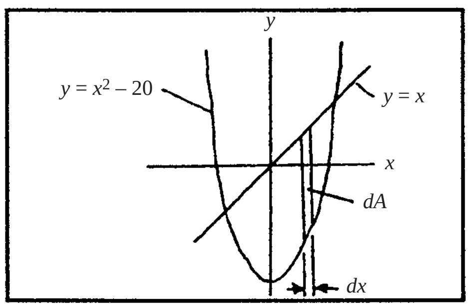
$$I_x = \frac{5\pi}{8} = 1.96\ \text{m}^4$$

From Problem 8.22, the area of the wing is $A = 7.85\ \text{m}^2$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1.96}{7.85}} \quad k_x = 0.500\ \text{m}$$

Problem 8.24 Determine I_y and k_y.





Solution: The straight line and curve intersect where $x = x^2 - 20$. Solving this equation for x, we obtain

$$x = \frac{1 \pm \sqrt{1+80}}{2} = -4, 5.$$

If we use a vertical strip: the area

$$dA = [x - (x^2 - 20)]\,dx.$$

Therefore

$$I_y = \int_A x^2\,dA = \int_{-4}^5 x^2(x - x^2 + 20)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5} + \frac{20x^3}{3}\right]_{-4}^5 = 522.$$

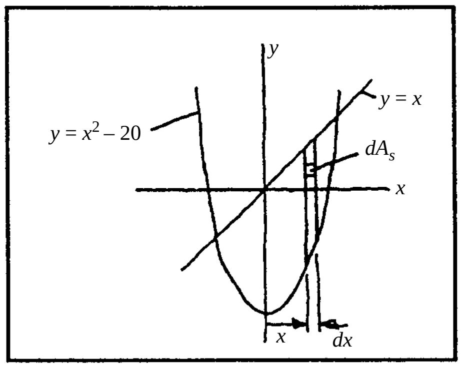
The area is

$$A = \int_A dA = \int_{-4}^5 (x - x^2 + 20)\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} + 20x\right]_{-4}^5 = 122.$$

So $k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{522}{122}} = 2.07.$

Problem 8.25 Determine I_x and k_x for the area in Problem 8.24.



Solution: Let us determine the moment of inertia about the x axis of a vertical strip holding x and dx fixed:

$$(I_x)_{(\text{strip})} = \int_{A_s} y^2\,dA_s = \int_{x^2-20}^{x} y^2(dx\,dy) = dx\left[\frac{y^3}{3}\right]_{x^2-20}^{x}$$

$$= \frac{dx}{3}(-x^6 + 60x^4 + x^3 - 1200x^2 + 8000).$$

Integrating this value from $x = -4$ to $x = 5$ (see the solution to Problem 8.24), we obtain I_x for the entire area:

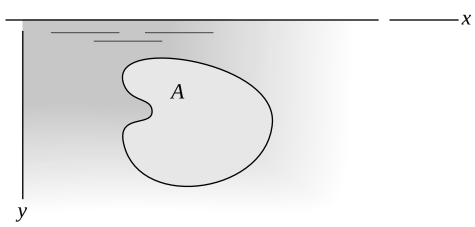
$$I_x = \int_{-4}^{5} \frac{1}{3}(-x^6 + 60x^4 + x^3 - 1200x^2 + 8000)\,dx$$

$$= \left[-\frac{x^7}{21} + 4x^5 + \frac{x^4}{12} - \frac{400x^3}{3} + \frac{8000x}{3}\right]_{-4}^{5} = 10{,}900.$$

From the solution to Problem 8.24, $A = 122$ so

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{10{,}900}{122}} = 9.45.$$

Problem 8.26 A vertical plate of area A is beneath the surface of a stationary body of water. The pressure of the water subjects each element dA of the surface of the plate to a force $(p_0 + \gamma y)\,dA$, where p_0 is the pressure at the surface of the water and γ is the weight density of the water. Show that the magnitude of the moment about the x axis due to the pressure on the front face of the plate is



$$M_{x\text{ axis}} = p_0\overline{y}A + \gamma I_x,$$

where $\overline{y}$ is the y coordinate of the centroid of A and I_x is the moment of inertia of A about the x axis.

Solution: The moment about the x axis is $dM = y(p_0 + \gamma y)\,dA$ integrating over the surface of the plate:

$$M = \int_A (p_0 + \gamma y)y\,dA.$$

Noting that p_0 and γ are constants over the area,

$$M = p_0\int_A y\,dA + \gamma\int y^2\,dA.$$

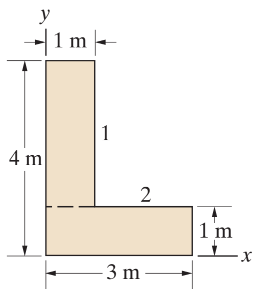
By definition,

$$\overline{y} = \frac{\int_A y\,dA}{A}$$

and $I_x = \int_A y^2\,dA$,

then $M = p_0\overline{y}A + \gamma I_X$, which demonstrates the result.

Problem 8.27 Using the procedure described in Active Example 8.3, determine I_x and k_x for the composite area by dividing it into rectangles 1 and 2 as shown.



Solution: Using results from Appendix B and applying the parallel-axis theorem, the moment on inertia about the x axis for area 1 is

$$(I_x)_1 = I_x + d_y^2 A = \frac{1}{12}(1\text{ m})(3\text{ m})^3 + (2.5\text{ m})[(1\text{ m})(3\text{ m})]$$

$$= 21.0\text{ m}^4$$

The moment of inertia about the x axis for area 2 is

$$(I_x)_2 = \frac{1}{3}(3\text{ m})(1\text{ m})^3 = 1\text{ m}^4.$$

The moment of inertia about the x axis for the composite area is

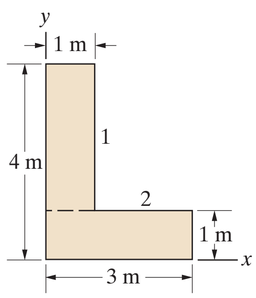
$$I_x = (I_x)_1 + (I_x)_2 = 22.0\text{ m}^4.$$

The radius of gyration about the x axis is

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{22.0\text{ m}^4}{6\text{ m}^2}} = 1.91\text{ m}$$

$\boxed{I_x = 22.0\text{ m}^4, k_x = 1.91\text{ m.}}$

Problem 8.28 Determine I_y and k_y for the composite area by dividing it into rectangles 1 and 2 as shown.



Solution: Using results from Appendix B, the moment of inertia about the y axis for area 1 is

$$(I_y)_1 = \frac{1}{3}(3\text{ m})(1\text{ m})^3 = 1\text{ m}^4.$$

The moment of inertia about the y axis for area 2 is

$$(I_y)_2 = \frac{1}{3}(1\text{ m})(3\text{ m})^3 = 9\text{ m}^4.$$

The moment of inertia about the y axis for the composite area is

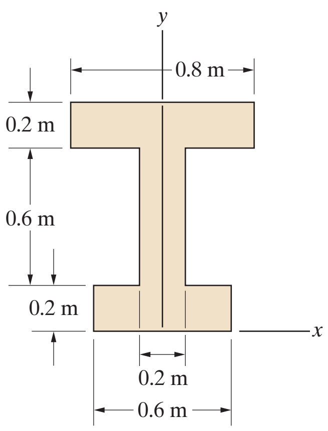
$$I_y = (I_y)_1 + (I_y)_2 = 10\text{ m}^4.$$

The radius of gyration about the y axis is

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{10\text{ m}^4}{6\text{ m}^2}} = 1.29\text{ m}$$

$\boxed{I_y = 10\text{ m}^4, k_x = 1.29\text{ m.}}$

Problem 8.29 Determine I_x and k_x.



Solution: Break into 3 rectangles

$$I_x = \frac{1}{3}(0.6)(0.2)^3 + \left[\frac{1}{12}(0.2)(0.6)^3 + (0.2)(0.6)(0.5)^2\right]$$

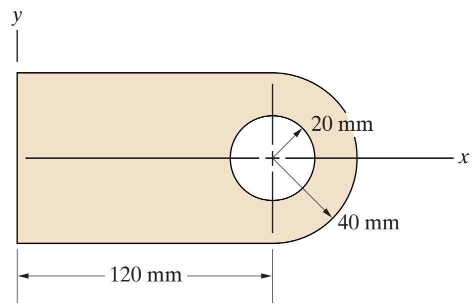
$$+ \frac{1}{12}(0.8)(0.2)^3 + (0.8)(0.2)(0.9)^2 = 0.1653 \text{ m}^4$$

$$A = (0.2)(0.6) + (0.6)(0.2) + (0.8)(0.2) = 0.4 \text{ m}^2$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.1653 \text{ m}^4}{0.4 \text{ m}^2}} = 0.643 \text{ m}$$

$$\Rightarrow \boxed{\begin{array}{l} I_x = 0.1653 \text{ m}^4 \\ k_x = 0.643 \text{ m} \end{array}}$$

Problem 8.30 In Example 8.4, determine I_x and k_x for the composite area.



Solution: The area is divided into a rectangular area *without the cutout* (part 1), a semicircular areas *without the cutout* (part 2), and the circular cutout (part 3).

Using the results from Appendix B, the moment of inertia of part 1 about the x axis is

$$(I_x)_1 = \frac{1}{12}(120 \text{ mm})(80 \text{ mm})^3 = 5.12 \times 10^6 \text{ mm}^4,$$

the moment of inertia of part 2 is

$$(I_x)_2 = \frac{1}{8}\pi(40 \text{ mm})^4 = 1.01 \times 10^6 \text{ mm}^4,$$

and the moment of inertia of part 3 is

$$(I_x)_3 = \frac{1}{4}\pi(20 \text{ mm})^4 = 1.26 \times 10^5 \text{ mm}^4.$$

The moment of inertia of the composite area is

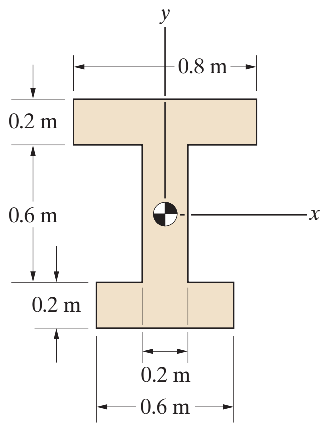
$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = 6.00 \times 10^6 \text{ mm}^4.$$

From Example 8.4, the composite area is $A = 1.086 \times 10^4 \text{ mm}^4$, so the radius of gyration about the x axis is

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6.00 \times 10^6 \text{ mm}^4}{1.086 \times 10^4 \text{ mm}^2}} = 23.5 \text{ mm}.$$

$\boxed{I_x = 6.00 \times 10^6 \text{ mm}^4, k_x = 23.5 \text{ mm.}}$

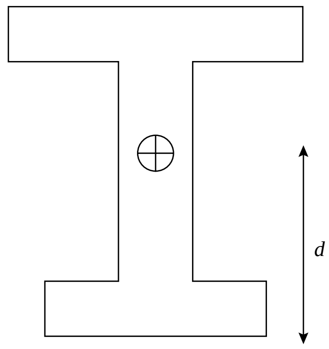
Problem 8.31 Determine I_x and k_x.



Solution: Break into 3 rectangles — See 8.29

First locate the centroid

$$d = \frac{(0.6)(0.2)(0.1) + (0.2)(0.6)(0.5) + (0.8)(0.2)(0.9)}{(0.6)(0.2) + (0.2)(0.6) + (0.8)(0.2)} = 0.54 \text{ m}$$



$$\boxed{\begin{aligned} I_x &= \left[\frac{1}{12}(0.2)(0.6)^3 + (0.2)(0.6)(d-0.5)^2\right] \\ &\quad + \left[\frac{1}{12}(0.6)(0.2)^3 + (0.6)(0.2)(d-0.1)^2\right] \\ &\quad + \left[\frac{1}{12}(0.8)(0.2)^3 + (0.8)(0.2)(0.9-d)^2\right] = 0.0487 \text{ m}^4 \\ k_x &= \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.0487 \text{ m}^4}{0.4 \text{ m}^2}} = 0.349 \text{ m} \end{aligned}}$$

Problem 8.32 Determine I_y and k_y.

Solution: Break into 3 rectangles — See 8.29

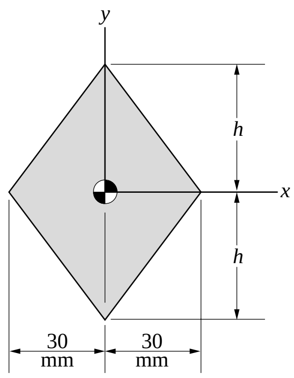
$$\boxed{\begin{aligned} I_y &= \frac{1}{12}(0.6)(0.2)^3 + \frac{1}{12}(0.2)(0.6)^3 + \frac{1}{12}(0.2)(0.8)^3 \\ &= 0.01253 \text{ m}^4 \\ k_y &= \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.01253 \text{ m}^4}{0.4 \text{ m}^2}} = 0.1770 \text{ m} \end{aligned}}$$

Problem 8.33 Determine J_O and k_O.

Solution: See 8.29, 8.31 and 8.32

$$\boxed{\begin{aligned} J_O &= I_x + I_y = 0.0612 \text{ m}^4 \\ K_O &= \sqrt{\frac{J_O}{A}} = \sqrt{\frac{0.0612 \text{ m}^4}{0.4 \text{ m}^2}} = 0.391 \text{ m} \end{aligned}}$$

Problem 8.34 If you design the beam cross section so that $I_x = 6.4 \times 10^5$ mm^4, what are the resulting values of I_y and J_O?



Solution: The area moment of inertia for a triangle about the base is

$$I_x = \left(\frac{1}{12}\right) bh^3,$$

from which $I_x = 2\left(\frac{1}{12}\right)(60)h^3 = 10h^3$ mm^4,

$$I_x = 10h^3 = 6.4 \times 10^5 \text{ mm}^4,$$

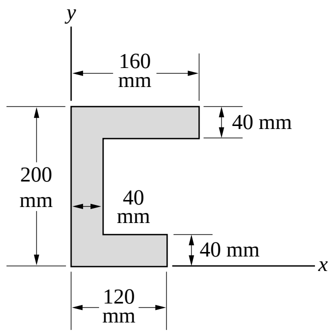
from which $h = 40$ mm.

$$I_y = 2\left(\frac{1}{12}\right)(2h)(30^3) = \left(\frac{1}{3}\right)h(30^3)$$

from which $I_y = \left(\frac{1}{3}\right)(40)(30^3) = 3.6 \times 10^5$ mm^4

and $J_O = I_x + I_y = 3.6 \times 10^5 + 6.4 \times 10^5 = 1 \times 10^6$ mm^4

Problem 8.35 Determine I_y and k_y.



Solution: Divide the area into three parts:

Part (1): The top rectangle.

$$A_1 = 160(40) = 6.4 \times 10^3 \text{ mm}^2,$$

$$d_{x1} = \frac{160}{2} = 80 \text{ mm},$$

$$I_{yy1} = \left(\frac{1}{12}\right)(40)(160^3) = 1.3653 \times 10^7 \text{ mm}^4.$$

From which

$$I_{y1} = d_{x1}^2 A_1 + I_{yy1} = 5.4613 \times 10^7 \text{ mm}^4.$$

Part (2): The middle rectangle:

$$A_2 = (200 - 80)(40) = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{x2} = 20 \text{ mm},$$

$$I_{yy2} = \left(\frac{1}{12}\right)(120)(40^3) = 6.4 \times 10^5 \text{ mm}^4.$$

From which,

$$I_{y2} = d_{x2}^2 A_2 + I_{yy2} = 2.56 \times 10^6 \text{ mm}^4.$$

Part (3) The bottom rectangle:

$$A_3 = 120(40) = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{x3} = \frac{120}{2} = 60 \text{ mm},$$

$$I_{yy3} = \left(\frac{1}{12}\right)40(120^3) = 5.76 \times 10^6 \text{ mm}^4$$

From which

$$I_{y3} = d_{X3}^2 A_3 + I_{yy3} = 2.304 \times 10^7 \text{ mm}^4$$

The composite:

$$I_y = I_{y1} + I_{y2} + I_{y3} = 8.0213 \times 10^7 \text{ mm}^4$$

$$k_y = \sqrt{\frac{I_y}{(A_1 + A_2 + A_3)}} = 70.8 \text{ mm}.$$

Problem 8.36 Determine I_x and k_x.

Solution: Use the solution to Problem 8.35. Divide the area into three parts:

Part (1): The top rectangle.

$$A_1 = 6.4 \times 10^3 \text{ mm}^2,$$

$$d_{y1} = 200 - 20 = 180 \text{ mm},$$

$$I_{xx1} = \left(\frac{1}{12}\right)(160)(40^3) = 8.533 \times 10^5 \text{ mm}^4.$$

From which

$$I_{x1} = d_{y1}^2 A_1 + I_{xx1} = 2.082 \times 10^8 \text{ mm}^4$$

Part (2): The middle rectangle:

$$A_2 = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{y2} = \frac{120}{2} + 40 = 100 \text{ mm},$$

$$I_{xx2} = \left(\frac{1}{12}\right)(40)(120^3) = 5.76 \times 10^6 \text{ mm}^4$$

from which

$$I_{x2} = d_{y2}^2 A_2 + I_{xx2} = 5.376 \times 10^7 \text{ mm}^4$$

Part (3) The bottom rectangle:

$$A_3 = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{y3} = 20 \text{ mm},$$

$$I_{xx3} = \left(\frac{1}{12}\right)120(40^3) = 6.4 \times 10^5 \text{ mm}^4$$

and $I_{x3} = d_{y3}^2 A_3 + I_{xx3} = 2.56 \times 10^6 \text{ mm}^4$.

The composite:

$$I_x = I_{x1} + I_{x2} + I_{x3} = 2.645 \times 10^8 \text{ mm}^4$$

$$k_x = \sqrt{\frac{I_x}{(A_1 + A_2 + A_3)}} = 128.6 \text{ mm}$$

Problem 8.37 Determine I_{xy}.

Solution: (See figure in Problem 8.35). Use the solutions in Problems 8.35 and 8.36. Divide the area into three parts:

Part (1): $A_1 = 160(40) = 6.4 \times 10^3 \text{ mm}^2$,

$$d_{x1} = \frac{160}{2} = 80 \text{ mm},$$

$$d_{y1} = 200 - 20 = 180 \text{ mm},$$

$$I_{xxyy1} = 0,$$

from which

$$I_{xy1} = d_{x1} d_{y1} A_1 + I_{xxyy1} = 9.216 \times 10^7 \text{ mm}^4.$$

Part (2) $A_2 = (200 - 80)(40) = 4.8 \times 10^3 \text{ mm}^2$,

$$d_{x2} = 20 \text{ mm},$$

$$d_{y2} = \frac{120}{2} + 40 = 100 \text{ mm},$$

from which

$$I_{xy2} = d_{x2} d_{y2} A_2 = 9.6 \times 10^6 \text{ mm}^4.$$

Part (3): $A_3 = 120(40) = 4.8 \times 10^3 \text{ mm}^2$,

$$d_{x3} = \frac{120}{2} = 60 \text{ mm},$$

$$d_{y3} = 20 \text{ mm},$$

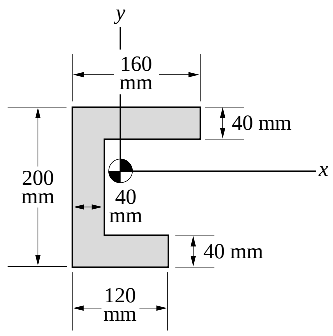
from which

$$I_{xy3} = d_{x3} d_{y3} A_3 = 5.76 \times 10^6.$$

The composite:

$$I_{xy} = I_{xy1} + I_{xy2} + I_{xy3} = 1.0752 \times 10^8 \text{ mm}^4$$

Problem 8.38 Determine I_x and k_x.



Solution: The strategy is to use the relationship $I_x = d^2A + I_{xc}$, where I_{xc} is the area moment of inertia about the centroid. From this $I_{xc} = -d^2A + I_x$. Use the solutions to Problems 8.35, 8.36, and 8.37. Divide the area into three parts and locate the centroid relative to the coordinate system in the Problems 8.35, 8.36, and 8.37.

Part (1) $A_1 = 6.4 \times 10^3 \text{ mm}^2$,

$d_{y1} = 200 - 20 = 180$ mm.

Part (2) $A_2 = (200 - 80)(40) = 4.8 \times 10^3 \text{ mm}^2$,

$$d_{x1} = \frac{160}{2} = 80 \text{ mm}, \quad d_{x2} = 20 \text{ mm},$$

$$d_{y2} = \frac{120}{2} + 40 = 100 \text{ mm},$$

Part (3) $A_3 = 120(40) = 4.8 \times 10^3 \text{ mm}^2$,

$$d_{x3} = \frac{120}{2} = 60 \text{ mm}, \quad d_{y3} = 20 \text{ mm}.$$

The total area is

$$A = A_1 + A_2 + A_3 = 1.6 \times 10^4 \text{ mm}^2.$$

The centroid coordinates are

$$\mathbf{x} = \frac{A_1 d_{x1} + A_2 d_{x2} + A_3 d_{x3}}{A} = 56 \text{ mm},$$

$$\mathbf{y} = \frac{A_1 d_{y1} + A_2 d_{y2} + A_3 d_{y3}}{A} = 108 \text{ mm}$$

from which

$$I_{xc} = -\mathbf{y}^2 A + I_x = -1.866 \times 10^8 + 2.645 \times 10^8$$

$$= 7.788 \times 10^7 \text{ mm}^4$$

$$k_{xc} = \sqrt{\frac{I_{xc}}{A}} = 69.77 \text{ mm}$$

Problem 8.39 Determine I_y and k_y.

Solution: The strategy is to use the relationship $I_y = d^2A + I_{yc}$, where I_{yc} is the area moment of inertia about the centroid. From this $I_{yc} = -d^2A + I_y$. Use the solution to Problem 8.38. The centroid coordinates are $\mathbf{x} = 56$ mm, $\mathbf{y} = 108$ mm, from which

$$I_{yc} = -\mathbf{x}^2 A + I_y = -5.0176 \times 10^7 + 8.0213 \times 10^7$$

$$= 3.0 \times 10^7 \text{ mm}^4,$$

$$k_{yc} = \sqrt{\frac{I_{yc}}{A}} = 43.33 \text{ mm}$$

Problem 8.40 Determine I_{xy}.

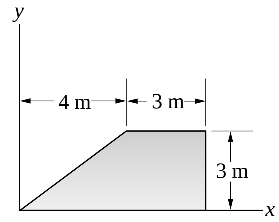
Solution: Use the solution to Problem 8.37. The centroid coordinates are

$$\mathbf{x} = 56 \text{ mm}, \quad \mathbf{y} = 108 \text{ mm},$$

from which $I_{xyc} = -\mathbf{xy}A + I_{xy} = -9.6768 \times 10^7 + 1.0752 \times 10^8$

$$= 1.0752 \times 10^7 \text{ mm}^4$$

Problem 8.41 Determine I_x and k_x.



Solution: Divide the area into two parts:

Part (1): a triangle and Part (2): a rectangle. The area moment of inertia for a triangle about the base is

$$I_x = \left(\frac{1}{12}\right) bh^3.$$

The area moment of inertia about the base for a rectangle is

$$I_x = \left(\frac{1}{3}\right) bh^3.$$

Part (1) $I_{x1} = \left(\frac{1}{12}\right) 4(3^3) = 9 \text{ m}^2.$

Part (2) $I_{x2} = \left(\frac{1}{3}\right) 3(3^3) = 27.$

The composite: $I_x = I_{x1} + I_{x2} = 36 \text{ m}^4$. The area:

$$A = \left(\frac{1}{2}\right) 4(3) + 3(3) = 15 \text{ m}^4.$$

$$k_x = \sqrt{\frac{I_x}{A}} = 1.549 \text{ m}.$$

Problem 8.42 Determine J_O and k_O.

Solution: (See Figure in Problem 8.41.) Use the solution to Problem 8.41.

Part (1): The area moment of inertia about the centroidal axis parallel to the base for a triangle is

$$I_{yc} = \left(\frac{1}{36}\right) bh^3 = \left(\frac{1}{36}\right) 3(4^3) = 5.3333 \text{ m}^4,$$

from which

$$I_{y1} = \left(\frac{8}{3}\right)^2 A_1 + I_{yc} = 48 \text{ m}^4.$$

where $A_1 = 6 \text{ m}^2$.

Part (2): The area moment of inertia about a centroid parallel to the base for a rectangle is

$$I_{yc} = \left(\frac{1}{12}\right) bh^3 = \left(\frac{1}{12}\right) 3(3^3) = 6.75 \text{ m}^4,$$

from which

$$I_{y2} = (5.5)^2 A_2 + I_{yc} = 279 \text{ m}^4,$$

where $A_2 = 9 \text{ m}^2$.

The composite: $I_y = I_{y1} + I_{y2} = 327 \text{ m}^4$, from which, using a result from Problem 8.41,

$$J_O = I_x + I_y = 327 + 36 = 363 \text{ m}^4$$

and $k_O = \sqrt{\dfrac{J_O}{A}} = 4.92 \text{ m}$

Problem 8.43 Determine I_{xy}.

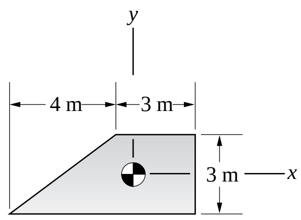
Solution: (See Figure in Problem 8.41.) Use the results of the solutions to Problems 8.41 and 8.42. The area cross product of the moment of inertia about centroidal axes parallel to the bases for a triangle is $I_{x'y'} = \frac{1}{72}b^2h^2$, and for a rectangle it is zero. Therefore:

$$I_{xy1} = \left(\frac{1}{72}\right)(4^2)(3^2) + \left(\frac{8}{3}\right)\left(\frac{3}{3}\right)A_1 = 18 \text{ m}^4$$

and $I_{xy2} = (1.5)(5.5)A_2 = 74.25 \text{ m}^4$,

$$I_{xy} = I_{x'y'1} + I_{xy2} = 92.25 \text{ m}^4$$

Problem 8.44 Determine I_x and k_x.



Solution: Use the results of Problems 8.41, 8.42, and 8.43. The strategy is to use the parallel axis theorem and solve for the area moment of inertia about the centroidal axis. The centroidal coordinate

$$\mathbf{y} = \frac{A_1(1) + A_2(1.5)}{A} = 1.3 \text{ m}.$$

From which

$$I_{xc} = -\mathbf{y}^2 A + I_x = 10.65 \text{ m}^4$$

and $k_{xc} = \sqrt{\frac{I_{xc}}{A}} = 0.843 \text{ m}$

Problem 8.45 Determine J_O and k_O.

Solution: Use the results of Problems 8.41, 8.42, and 8.43. The strategy is to use the parallel axis theorem and solve for the area moment of inertia about the centroidal axis. The centroidal coordinate:

$$\mathbf{x} = \frac{A_1\left(\frac{8}{3}\right) + A_2(5.5)}{A} = 4.3667 \text{ m},$$

from which

$$I_{YC} = -\mathbf{x}^2 A + I_Y = 40.98 \text{ m}^4.$$

Using a result from Problem 8.44,

$$J_O = I_{XC} + I_{YC} = 10.65 + 40.98 = 51.63 \text{ m}^4$$

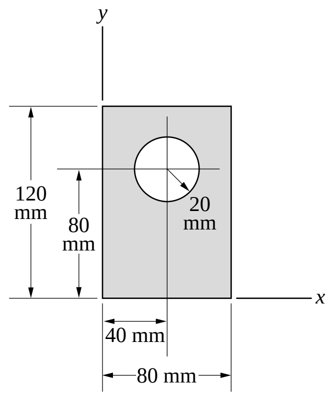
and $k_O = \sqrt{\frac{J_O}{A}} = 1.855 \text{ m}$

Problem 8.46 Determine I_{xy}.

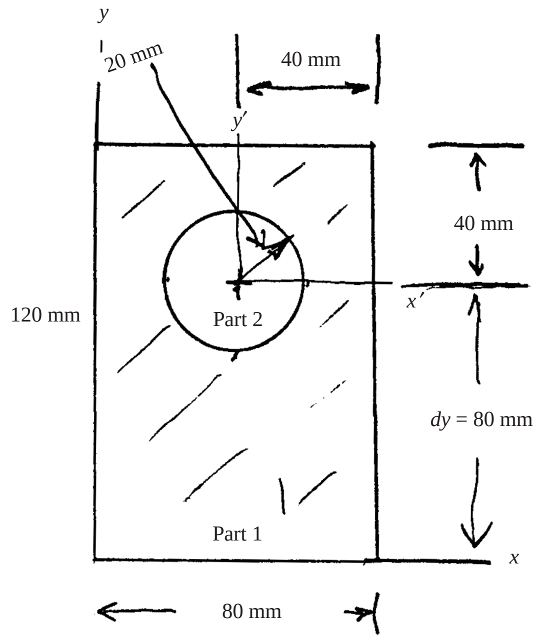
Solution: Use the results of Problems 8.41–8.45. The strategy is to use the parallel axis theorem and solve for the area moment of inertia about the centroidal axis. Using the centroidal coordinates determined in Problems 8.44 and 8.45,

$$I_{xy} = -\mathbf{xy}A + I_{xy} = -85.15 + 92.25 = 7.1 \text{ m}^4$$

Problem 8.47 Determine I_x and k_x.



Solution: Let Part 1 be the entire rectangular solid without the hole and let part 2 be the hole.



$$\text{Area} = hb - \pi R^2 = (80)(120) - \pi R^2$$

$$I_{x1} = \tfrac{1}{3}bh^3 \quad \text{where } b = 80 \text{ mm}$$

$$h = 120 \text{ mm}$$

$$I_{x1} = \tfrac{1}{3}(80)(120)^3 = 4.608 \times 10^7 \text{ mm}^4$$

For Part 2,

$$I_{x'2} = \tfrac{1}{4}\pi R^4 = \tfrac{1}{4}\pi(20)^4 \text{ mm}^4$$

$$I_{x'2} = 1.257 \times 10^5 \text{ mm}^4$$

$$I_{x2} = I_{x'2} + d_y^2 A$$

where $A = \pi R^2 = 1257 \text{ mm}^2$

$d = 80$ mm

$$I_{x2} = 1.257 \times 10^5 + \pi(20)^2(80)^2$$

$$I_{x2} = 0.126 \times 10^6 + 8.042 \times 10^6 \text{ mm}^4$$

$$= 8.168 \times 10^6 \text{ mm}^4 = 0.817 \times 10^7 \text{ mm}^4$$

$$I_x = I_{x1} - I_{x2} = 3.79 \times 10^7 \text{ mm}^4$$

$$\text{Area} = 8343 \text{ mm}^2$$

$$k_x = \sqrt{\frac{I_x}{\text{Area}}} = 67.4 \text{ mm}$$

Problem 8.48 Determine J_O and k_O.

Solution: For the rectangle,

$J_{O1} = I_{x1} + I_{y1} = \frac{1}{3}bh^3 + \frac{1}{3}hb^3$

$J_{O1} = 4.608 \times 10^7 + 2.048 \times 10^7 \text{ mm}^4$

$J_{O1} = 6.656 \times 10^7 \text{ mm}^4$

$A_1 = bh = 9600 \text{ mm}^2$

For the circular cutout about $x'y'$

$J'_{O2} = I_{x'2} + I_{y'_2} = \frac{1}{4}\pi R^4 + \frac{1}{4}\pi R^4$

$J'_{O2} = 1.257 \times 10^5 + 1.257 \times 10^5 \text{ mm}^4$

$J'_{O2} = 2.513 \times 10^5 \text{ mm}^2$

Using the parallel axis theorem to determine J_{O2} (about x, y)

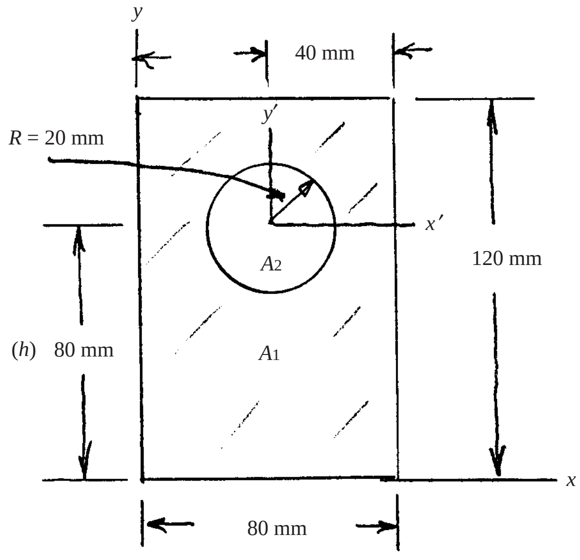
$J_{O2} = J'_{0_2} + (d_x^2 + d_y^2)A_2$

$A_2 = \pi R^2 = 1257 \text{ mm}^2$

$J_{O2} = 1.030 \times 10^7 \text{ mm}^4$

$J_O = J_{O1} - J_{O2}$

$J_O = 6.656 \times 10^7 - 1.030 \times 10^7 \text{ mm}^4$



(*b*)

$J_O = 5.63 \times 10^7 \text{ mm}^4$

$k_O = \sqrt{\dfrac{J_O}{\text{Area}}} = \sqrt{\dfrac{J_O}{A_1 - A_2}}$

$k_O = 82.1 \text{ mm}$

Problem 8.49 Determine I_{xy}.

Solution:

$A_1 = (80)(120) = 9600 \text{ mm}^2$

$A_2 = \pi R^2 = \pi(20)^2 = 1257 \text{ mm}^2$

For the rectangle (A_1)

$I_{xy_1} = \frac{1}{4}b^2h^2 = \frac{1}{4}(80)^2(120)^2$

$I_{xy_1} = 2.304 \times 10^7 \text{ mm}^2$

For the cutout

$I_{x'y'2} = 0$

and by the parallel axis theorem

$I_{xy2} = I_{x'y'2} + A_2(d_x)(d_y)$

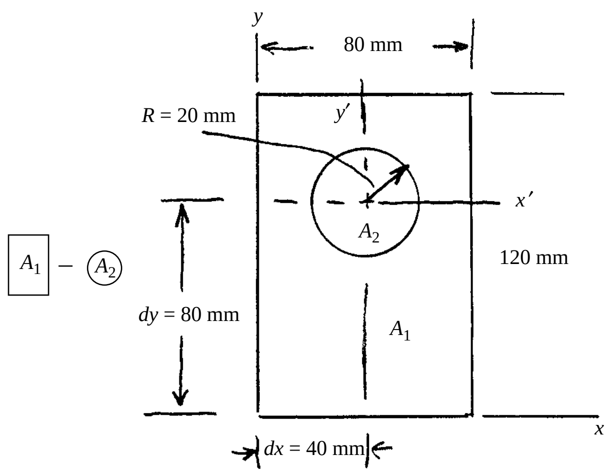
$I_{xy2} = 0 + (1257)(40)(80)$

$I_{xy2} = 4.021 \times 10^6 \text{ mm}^4$

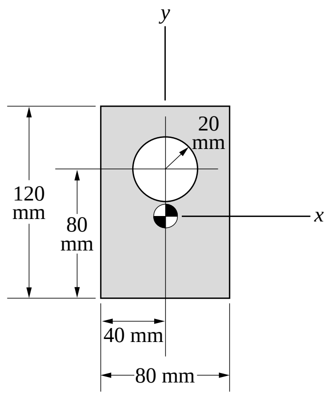
$I_{xy} = I_{xy1} - I_{xy2}$

$I_{xy} = 2.304 \times 10^7 - 0.402 \times 10^7 \text{ mm}^4$

$I_{xy} = 1.90 \times 10^7 \text{ mm}^4$

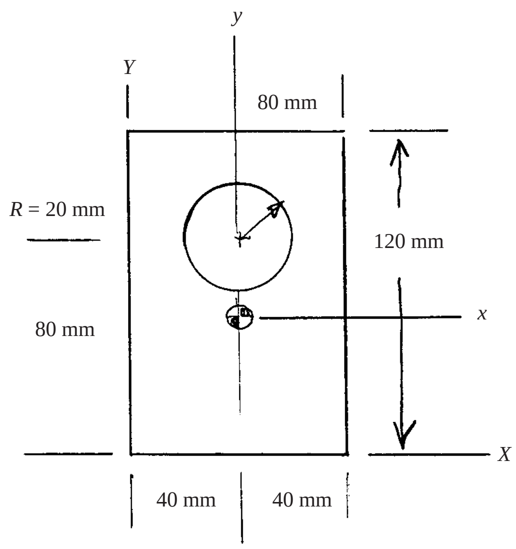


Problem 8.50 Determine I_x and k_x.



Solution: We must first find the location of the centroid of the total area. Let us use the coordinates XY to do this. Let A_1 be the rectangle and A_2 be the circular cutout. Note that by symmetry $X_c = 40$ mm

	Area	X_c	Y_c
Rectangle$_1$	9600 mm^2	40 mm	60 mm
Circle$_2$	1257 mm^2	40 mm	80 mm



$A_1 = 9600 \text{ mm}^2$

$A_2 = 1257 \text{ mm}^2$

For the composite,

$$X_c = \frac{A_1 X_{c1} - A_2 X_{c2}}{A_1 - A_2} = 40 \text{ mm}$$

$$Y_c = \frac{A_1 Y_{c1} - A_2 Y_{c2}}{A_1 - A_2} = 57.0 \text{ mm}$$

Now let us determine I_x and k_x about the centroid of the composite body.

Rectangle about its centroid (40, 60) mm

$$I_{x1} = \frac{1}{12} bh^3 = \frac{1}{12}(80)(120)^3$$

$I_{x1} = 1.152 \times 10^7 \text{ mm}^3$,

Now to C

$I_{xc1} = I_{x1} + (60 - Y_c)^2 A_1$

$I_{xc1} = 1.161 \times 10^7 \text{ mm}^4$

Circular cut out about its centroid

$A_2 = \pi R^2 = (20)^2 \pi = 1257 \text{ mm}^2$

$I_{x2} = \frac{1}{4}\pi R^4 = \pi(20)^4/4$

$I_{x2} = 1.26 \times 10^5 \text{ mm}^4$

Now to $C \rightarrow d_{y2} = 80 - 57 = 23$ mm

$I_{xc2} = I_{x2} + (d_{y2})^2 A_2$

$I_{xc2} = 7.91 \times 10^5 \text{ mm}^4$

For the composite about the centroid

$I_x = I_{xc1} - I_{xc2}$

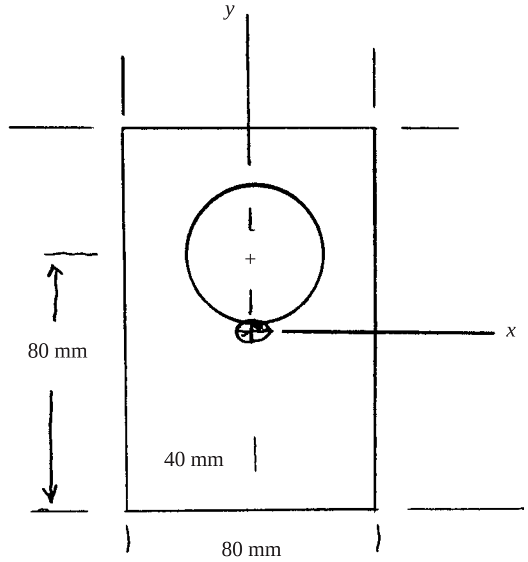
$I_x = 1.08 \times 10^7 \text{ mm}^4$

The composite Area $= 9600 - 1257 \text{ mm}^2$

$= 8343 \text{ mm}^2$

$$k_x = \sqrt{\frac{I_x}{A}} = 36.0 \text{ mm}$$

Problem 8.51 Determine I_y and k_y.



Solution: From the solution to Problem 8.50, the centroid of the composite area is located at (40, 57.0) mm.

The area of the rectangle, A_1, is 9600 mm^2.

The area of the cutout, A_2, is 1257 mm^2.

The area of the composite is 8343 mm^2.

(1) Rectangle about its centroid (40, 60) mm.

$$I_{y1} = \frac{1}{12}hb^3 = \frac{1}{12}(120)(80)^3$$

$$I_{y1} = 5.12 \times 10^6 \text{ mm}^4$$

$$d_{x1} = 0$$

(2) Circular cutout about its centroid (40, 80)

$$I_{y2} = \pi R^4/4 = 1.26 \times 10^5 \text{ mm}^4$$

$$d_{x2} = 0$$

Since d_{x1} and d_{x2} are zero. (no translation of axes in the x-direction), we get

$$I_y = I_{y1} - I_{y2}$$

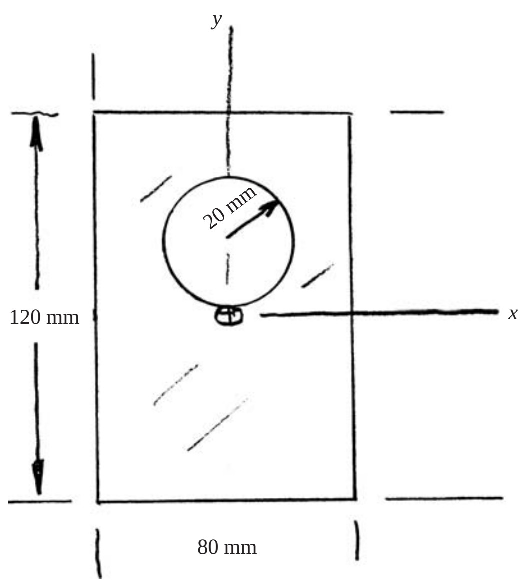
$$I_y = 4.99 \times 10^6 \text{ mm}^4$$

Finally,

$$k_y = \sqrt{\frac{I_y}{A_1 - A_2}} = \sqrt{\frac{4.99 \times 10^6}{8343}}$$

$$k_y = 24.5 \text{ mm}$$

Problem 8.52 Determine J_O and k_O.



Solution: From the solutions to Problems 8.51 and 8.52,

$$I_x = 1.07 \times 10^7 \text{ mm}^4$$

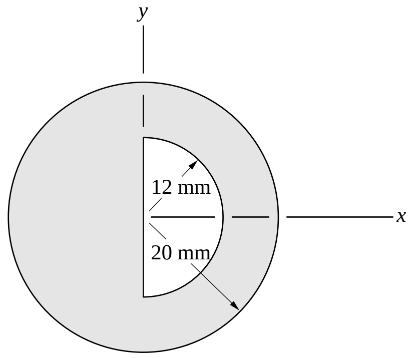
$$I_y = 4.99 \times 10^6 \text{ mm}^4$$

and $A = 8343$ mm^2

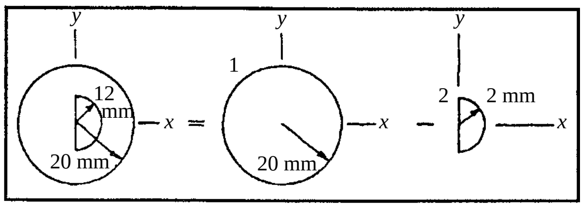
$$J_O = I_x + I_y = 1.57 \times 10^7 \text{ mm}^4$$

$$k_O = \sqrt{\frac{J_O}{A}} = 43.4 \text{ mm}$$

Problem 8.53 Determine I_y and k_y.



Solution: Treat the area as a circular area with a half-circular cutout: From Appendix *B*,



$$(I_y)_1 = \tfrac{1}{4}\pi(20)^4 \text{ mm}^4$$

and $(I_y)_2 = \tfrac{1}{8}\pi(12)^4 \text{ mm}^4$,

$$\text{so } I_y = \tfrac{1}{4}\pi(20)^4 - \tfrac{1}{8}\pi(12)^4 = 1.18 \times 10^5 \text{ mm}^4.$$

The area is $A = \pi(20)^2 - \tfrac{1}{2}\pi(12)^2 = 1030 \text{ mm}^2$

$$\text{so,} \quad k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1.18 \times 10^5}{1.03 \times 10^3}}$$

$$= 10.7 \text{ mm}$$

Problem 8.54 Determine J_O and k_O.

Solution: Treating the area as a circular area with a half-circular cutout as shown in the solution of Problem 8.53, from Appendix *B*,

$$(J_O)_1 = (I_x)_1 + (I_y)_1 = \tfrac{1}{2}\pi(20)^4 \text{ mm}^4$$

and $(J_O)_2 = (I_x)_2 + (I_y)_2 = \tfrac{1}{4}\pi(12)^4 \text{ mm}^4$.

Therefore $J_O = \tfrac{1}{2}\pi(20)^4 - \tfrac{1}{4}\pi(12)^4$

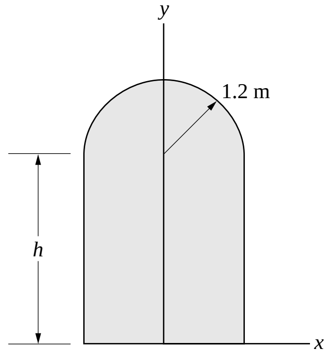
$$= 2.35 \times 10^5 \text{ mm}^4.$$

From the solution of Problem 8.53,

$$A = 1030 \text{ in}^2 \; R_o = \sqrt{\frac{J_O}{A}}$$

$$= \sqrt{\frac{2.35 \times 10^5}{1.03 \times 10^3}} = 15.1 \text{ mm}.$$

Problem 8.55 Determine I_y and k_y if $h = 3$ m.

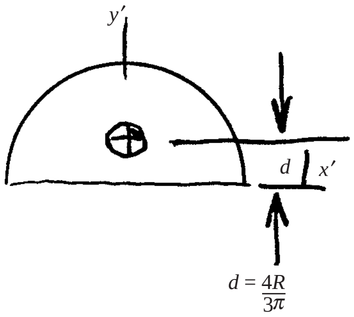


Solution: Break the composite into two parts, a rectangle and a semi-circle.

For the semi-circle

$$I_{x'c} = \left(\frac{\pi}{8} - \frac{9}{8\pi}\right) R^4$$

$$I_{y'c} = \frac{1}{8}\pi R^4 \quad d = \frac{4R}{3\pi}$$



To get moments about the x and y axes, the (d_{xc}, d_{yc}) for the semi-circle are

$$d_{xc} = 0, \quad d_{yc} = 3 \text{ m} + \frac{4R}{3\pi}$$

and $A_c = \pi R^2/2 = 2.26 \text{ m}^2$

$$I_{y'c} = \frac{1}{8}\pi R^4$$

and $I_{yc} = I_{y'c} + d_{xc}^2 A \quad (d_x = 0)$

$$I_{yc} = I_{y'c} = \pi(1.2)^4/8$$

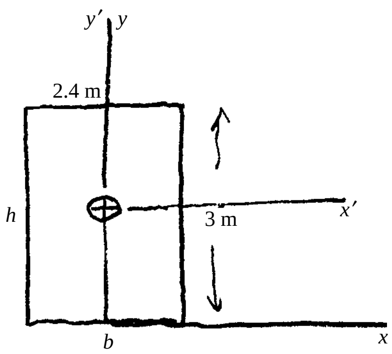
$$I_{yc} = 0.814 \text{ m}^4$$

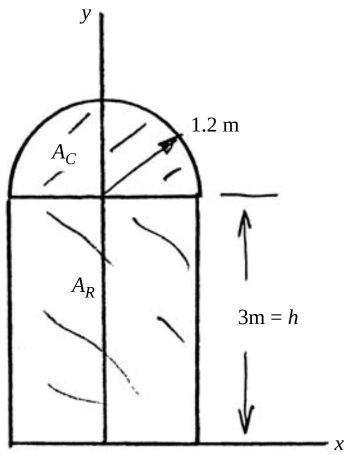
For the Rectangle

$$I_{x'R} = \frac{1}{12} bh^3$$

$$I_{y'R} = \frac{1}{12} hb^3$$

$$A_R = bh$$





To get moments of area about the x, y axes, $d_{xR} = 0$, $d_{yR} = 1.5$ m

$$I_{yR} = I_{y'R} + (\cancelto{0}{d_{xR}})^2(bh)$$

$$I_{yR} = I_{y'R} = \frac{1}{12}(3)(2,4)^3 \text{ m}^4$$

$$I_{yR} = 3.456 \text{ m}^2$$

$$A_R = bh = 7.2 \text{ m}^2$$

$$I_y = I_{yc} + I_{yR}$$

$$I_y = 4.27 \text{ m}^2$$

To find k_y, we need the total area, $A = A_R + A_c$

$$A = 7.20 + 2.26 \text{ m}^2$$

$$A = 9.46 \text{ m}^2$$

$$k_y = \sqrt{\frac{I_y}{A}} = 0.672 \text{ m}$$

Problem 8.56 Determine I_x and k_x if $h = 3$ m.

Solution: Break the composite into two parts, the semi-circle and the rectangle. From the solution to Problem 8.55,

$$I_{x'c} = \left(\frac{\pi}{8} - \frac{9}{8\pi}\right) R^4$$

$$d_{yc} = \left(3 + \frac{4R}{3\pi}\right) \text{ m}$$

$$A_c = 2.26 \text{ m}^2$$

$$I_{xc} = I_{x'c} + A_c d_{yc}^2$$

Substituting in numbers, we get

$$I_{x'c} = 0.0717 \text{ m}^4$$

$$d_{yc} = 3.509 \text{ m}$$

and $I_{xc} = I_{x'c} + A_c d_y^2$

$$I_{xc} = 27.928 \text{ m}^2$$

For the Rectangle $h = 3$ m, $b = 2.4$ m

Area: $A_R = bh = 7.20 \text{ m}^2$

$$I_{x'R} = \frac{1}{12} bh^3, \ d_{yR} = 1.5 \text{ m}$$

$$I_{xR} = I_{x'R} + d_{yR}^2 A_R$$

Substituting, we get

$$I_{x'R} = 5.40 \text{ m}^4$$

$$I_{xR} = 21.6 \text{ m}^4$$

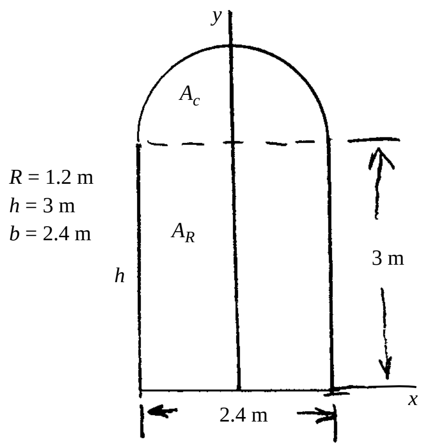
For the composite,

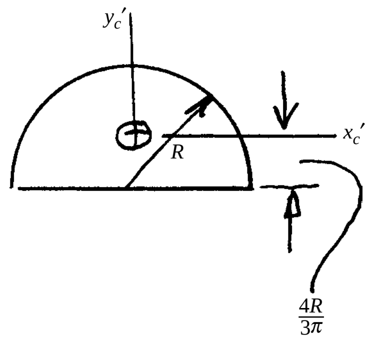
$$I_x = I_{xR} + I_{xc}$$

$$I_x = 49.5 \text{ m}^4$$

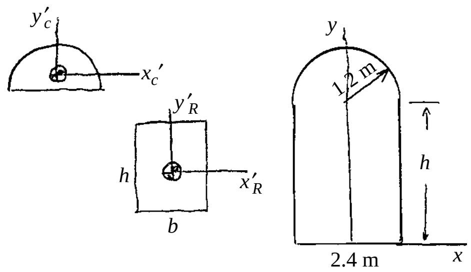
Also $k_x = \sqrt{\dfrac{I_x}{A_R + A_c}} = 2.29$ m

$$k_x = 2.29 \text{ m}$$





Problem 8.57 If $I_y = 5\text{ m}^4$, what is the dimension of h?



Solution: From the solution to Problem 8.55, we have:

For the semicircle

$I_{y'c} = I_y = \pi(1.2)^{4/8} = 0.814\text{ m}^2$

For the rectangle

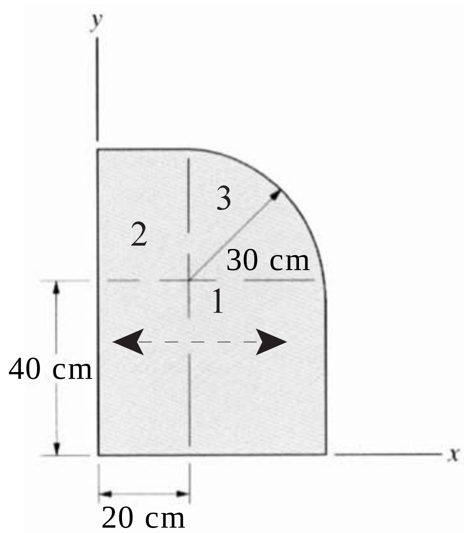
$I_{y'R} = I_{yR} = \dfrac{1}{12}(h)(2.4)^3\text{ m}^4$

Also, we know $I_{yR} + I_{yc} = 5\text{ m}^4$.

Hence $0.814 + \dfrac{1}{12}(h)(2.4)^3 = 5$

Solving, $h = 3.63$ m

Problem 8.58 Determine I_y and k_y.



Solution: Let the area be divided into parts as shown. The areas and the coordinates of their centroids are

$A_1 = (40)(50) = 2000\text{ cm}^2,\ \ \bar{x}_1 = 25\text{ cm},\ \ \bar{y}_1 = 20\text{ cm},$

$A_2 = (20)(30) = 600\text{ cm}^2,\ \ \bar{x}_2 = 10\text{ cm},\ \ \bar{y}_2 = 55\text{ cm},$

$A_3 = \dfrac{1}{4}\pi(30)^2 = 707\text{ cm}^2,$

$\bar{x}_3 = 20 + \dfrac{4(30)}{3\pi} = 32.7\text{ cm},\ \ \bar{y}_3 = 40 + \dfrac{4(30)}{3\pi} = 52.7\text{ cm}.$

Using the results from Appendix B, the moments of inertia of the parts about the y axis are

$(I_y)_1 = \dfrac{1}{3}(40\text{ cm})(50\text{ cm})^3 = 167 \times 10^4\text{ cm}^4,$

$(I_y)_2 = \dfrac{1}{3}(30\text{ cm})(20\text{ cm})^3 = 8.00 \times 10^4\text{ cm}^4,$

$$(I_y)_3 = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)(30\text{ cm})^4 + \left[20\text{ cm} + \frac{4(30\text{ cm})}{3\pi}\right]^2\left[\frac{\pi}{4}(30\text{ cm})^2\right]$$

$$= 80.2 \times 10^4\text{ cm}^4.$$

The moment of inertia of the composite area about the y axis is

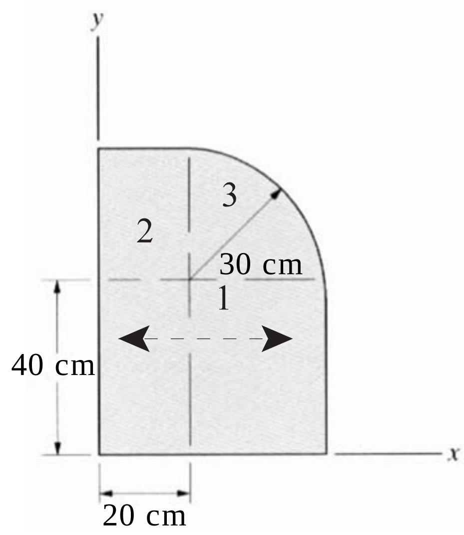
$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = 2.55 \times 10^6\text{ cm}^4.$$

The composite area is $A = A_1 + A_2 + A_3 = 3310\text{ cm}^2$.

The radius of gyration about the y axis is $k_y = \sqrt{\dfrac{I_y}{A}} = 27.8$ cm.

$\boxed{I_y = 2.55 \times 10^6\text{ cm}^4,\ \ k_y = 27.8\text{ cm.}}$

Problem 8.59 Determine I_x and k_x.



Solution: See the solution to Problem 8.58.
Let the area be divided into parts as shown. The areas and the coordinates of their centroids are

$A_1 = (40)(50) = 2000\ \text{cm}^2,\ \bar{x}_1 = 25\ \text{cm},\ \bar{y}_1 = 20\ \text{cm},$

$A_2 = (20)(30) = 600\ \text{cm}^2,\ \bar{x}_2 = 10\ \text{cm},\ \bar{y}_2 = 55\ \text{cm},$

$A_3 = \frac{1}{4}\pi(30)^2 = 707\ \text{cm}^2,\ \bar{x}_3 = 20 + \frac{4(30)}{3\pi} = 32.7\ \text{cm},$

$\bar{y}_3 = 40 + \frac{4(30)}{3\pi} = 52.7\ \text{cm}.$

Using the results from Appendix B, the moments of inertia of the parts about the x axis are

$(I_x)_1 = \frac{1}{3}(50\ \text{cm})(40\ \text{cm})^3 = 1.07 \times 10^6\ \text{cm}^4,$

$(I_x)_2 = \frac{1}{12}(20\ \text{cm})(30\ \text{cm})^3 + (55\ \text{cm})^2(600\ \text{cm}^2) = 1.86 \times 10^6\ \text{cm}^4,$

$(I_x)_3 = \left(\frac{\pi}{4} - \frac{4}{9\pi}\right)(30\ \text{cm})^4 + \left[40\ \text{cm} + \frac{4(30\ \text{cm})}{3\pi}\right]^2 \left[\frac{\pi}{4}(30\ \text{cm})^2\right] = 2.01 \times 10^6\ \text{cm}^4.$

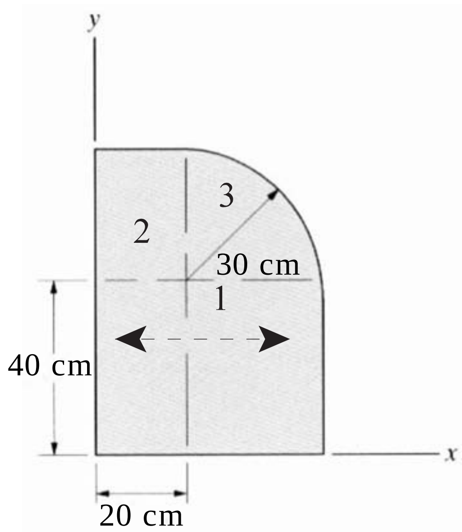
The moment of inertia of the composite area about the x axis is

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = 4.94 \times 10^6\ \text{cm}^4.$$

The composite area is $A = A_1 + A_2 + A_3 = 3310\ \text{cm}^2$.
The radius of gyration about the y axis is $k_x = \sqrt{\frac{I_x}{A}} = 38.6\ \text{cm}$.

$\boxed{I_x = 4.94 \times 10^6\ \text{cm}^4,\ k_x = 38.6\ \text{cm}}$

Problem 8.60 Determine I_{xy}.



Solution: See the solution to Problem 8.58.
Let the area be divided into parts as shown. The areas and the coordinates of their centroids are

$$A_1 = (40)(50) = 2000 \text{ cm}^2, \; \bar{x}_1 = 25 \text{ cm}, \; \bar{y}_1 = 20 \text{ cm},$$

$$A_2 = (20)(30) = 600 \text{ cm}^2, \; \bar{x}_2 = 10 \text{ cm}, \; \bar{y}_2 = 55 \text{ cm},$$

$$A_3 = \frac{1}{4}\pi(30)^2 = 707 \text{ cm}^2, \; \bar{x}_3 = 20 + \frac{4(30)}{3\pi} = 32.7 \text{ cm},$$

$$\bar{y}_3 = 40 + \frac{4(30)}{3\pi} = 52.7 \text{ cm}.$$

Using the results from Appendix B, the products of inertia of the parts about are

$$(I_{xy})_1 = \frac{1}{4}(50 \text{ cm})^2(40 \text{ cm})^2 = 10.0 \times 10^5 \text{ cm}^4,$$

$$(I_{xy})_2 = (10 \text{ cm})(55 \text{ cm})(600 \text{ cm}^2) = 3.30 \times 10^5 \text{ cm}^4,$$

$$(I_{xy})_3 = \left(\frac{1}{8} - \frac{4}{9\pi}\right)(30 \text{ cm})^4 + \left[20 \text{ cm} + \frac{4(30 \text{ cm})}{3\pi}\right]\left[40 \text{ cm} + \frac{4(30 \text{ cm})}{3\pi}\right][707 \text{ cm}^2]$$

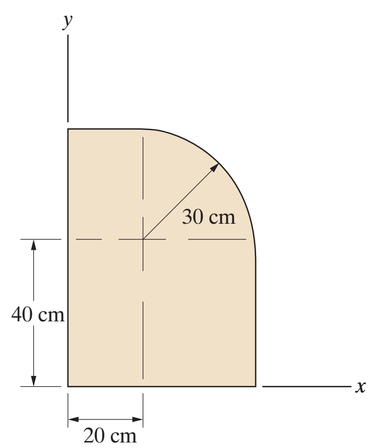
$$= 12.1 \times 10^5 \text{ cm}^4.$$

The product of inertia of the composite area is

$$I_{xy} = (I_{xy})_1 + (I_{xy})_2 + (I_{xy})_3 = 2.54 \times 10^6 \text{ cm}^4.$$

$\boxed{I_{xy} = 2.54 \times 10^6 \text{ cm}^4.}$

Problem 8.61 Determine I_y and k_y.



Solution: See the solution to Problem 8.58.
In terms of the coordinate system used in Problem 8.58, the areas and the coordinates of their centroids are

$$A_1 = (40)(50) = 2000 \text{ cm}^2, \; \bar{x}_1 = 25 \text{ cm}, \; \bar{y}_1 = 20 \text{ cm},$$

$$A_2 = (20)(30) = 600 \text{ cm}^2, \; \bar{x}_2 = 10 \text{ cm}, \; \bar{y}_2 = 55 \text{ cm},$$

$$A_3 = \frac{1}{4}\pi(30)^2 = 707 \text{ cm}^2, \; \bar{x}_3 = 20 + \frac{4(30)}{3\pi} = 32.7 \text{ cm},$$

$$\bar{y}_3 = 40 + \frac{4(30)}{3\pi} = 52.7 \text{ cm}.$$

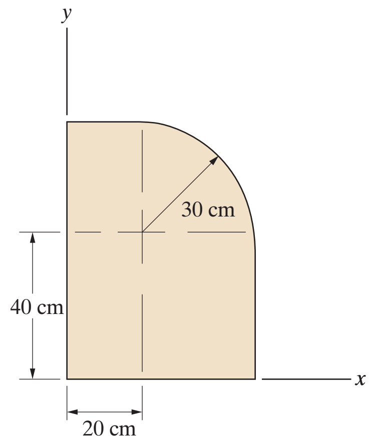
The composite area is $A = A_1 + A_2 + A_3 = 3310 \text{ cm}^2$.
The x coordinate of its centroid is $\bar{x} = \dfrac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A} = 23.9$ cm
The moment of inertia about the y axis in terms of the coordinate system used in Problem 8.58 is $I_y = 2.55 \times 10^6 \text{ cm}^4$. Applying the parallel axis theorem, the moment of inertia about the y axis through the centroid of the area is

$$I_y = 2.55 \times 10^6 \text{ cm}^4 - (23.9 \text{ cm})^2(3310 \text{ cm}^2) = 6\ 55 \times 10^5 \text{ cm}^4.$$

The radius of gyration about the y axis is $k_y = \sqrt{\dfrac{I_y}{A}} = 14.1$ cm.

$\boxed{I_y = 6.55 \times 10^5 \text{ cm}^4, \; k_y = 14.1 \text{ cm}.}$

Problem 8.62 Determine I_x and k_x.



Solution: See the solution to Problem 8.59.
In terms of the coordinate system used in Problem 8.59, the areas and the coordinates of their centroids are

$$A_1 = (40)(50) = 2000 \text{ cm}^2, \ \bar{x}_1 = 25 \text{ cm}, \ \bar{y}_1 = 20 \text{ cm},$$

$$A_2 = (20)(30) = 600 \text{ cm}^2, \ \bar{x}_2 = 10 \text{ cm}, \ \bar{y}_2 = 55 \text{ cm},$$

$$A_3 = \frac{1}{4}\pi(30)^2 = 707 \text{ cm}^2, \ \bar{x}_3 = 20 + \frac{4(30)}{3\pi} = 32.7 \text{ cm},$$

$$\bar{y}_3 = 40 + \frac{4(30)}{3\pi} = 52.7 \text{ cm}.$$

The composite area is $A = A_1 + A_2 + A_3 = 3310 \text{ cm}^2$.
The x coordinate of its centroid is $\bar{y} = \dfrac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A} = 33.3$ cm
The moment of inertia about the x axis in terms of the coordinate system used in Problem 8.59 is $I_x = 4.94 \times 10^6 \text{ cm}^4$. Applying the parallel axis theorem, the moment of inertia about the x axis through the centroid of the area is

$$I_x = 4.94 \times 10^6 \text{ cm}^4 - (33.3 \text{ cm})^2 (3310 \text{ cm}^2) = 1.26 \times 10^6 \text{ cm}^4.$$

The radius of gyration about the x axis is $k_x = \sqrt{\dfrac{I_x}{A}} = 19.5$ cm.

$$\boxed{I_x = 1.26 \times 10^6 \text{ cm}^4, \ k_x = 19.5 \text{ cm}.}$$

Problem 8.63 Determine I_{xy}.

Solution: See the solution to Problem 8.60.

$$I_{xy} = [0 + (1.0 \text{ m})(0.8 \text{ m})(d_x - 0.5 \text{ m})(d_y - 0.4 \text{ m})]$$

$$- [0 + \pi(0.2 \text{ m})^2(d_x - 0.4 \text{ m})(d_y - 0.3 \text{ m})]$$

$$+ \left[\frac{1}{24}(0.8 \text{ m})^2 \ (0.6 \text{ m})^2 - \frac{1}{2}(0.8 \text{ m})(0.6 \text{ m})(0.2 \text{ m})\left(\frac{0.8 \text{ m}}{3}\right)\right.$$

$$\left. + \frac{1}{2}(0.8 \text{ m})(0.6 \text{ m})(d_x - 1.2 \text{ m})\left(d_y - \frac{0.8 \text{ m}}{3}\right)\right]$$

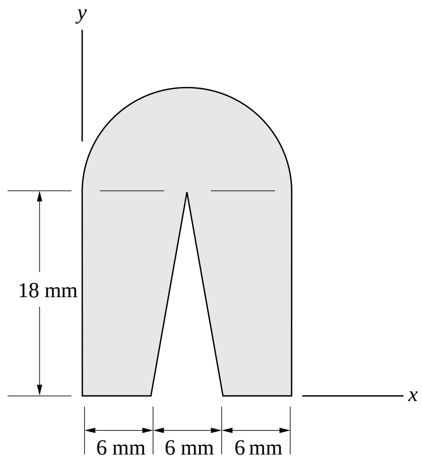
Solving: $I_{xy} = -0.0230 \text{ m}^4$

Check using the noncentroidal product of inertia from Problem 8.60 we have

$$I_{xy} = I_{xy'} - A d_x d_y = 0.2185 \text{ m}^4 - (0.914 \text{ m}^2)(0.697 \text{ m})(0.379 \text{ m})$$

$$= -0.0230 \text{ m}^4$$

Problem 8.64 Determine I_y and k_y.



Solution: Divide the area into three parts:

Part (1) The rectangle 18 by 18 mm; Part (2) The triangle with base 6 mm and altitude 18 mm; Part (3) The semicircle of 9 mm radius.

Part (1): $A_1 = 18(18) = 324 \text{ mm}^2$,

$\mathbf{x}_1 = 9 \text{ mm}$,

$\mathbf{y}_1 = 9 \text{ mm}$,

$$I_{xx1} = \left(\frac{1}{12}\right) 18(18^3) = 8748 \text{ mm}^4,$$

$$I_{yy1} = \left(\frac{1}{12}\right) 18(18^3) = 8748 \text{ mm}^4.$$

Part (2): $A_2 = \left(\frac{1}{2}\right) 18(6) = 54 \text{ mm}^2$,

$\mathbf{x}_2 = 9 \text{ mm}$,

$$\mathbf{y}_2 = \left(\frac{1}{3}\right) 18 = 6 \text{ mm},$$

$$I_{xx2} = \left(\frac{1}{36}\right) 6(18^3) = 972 \text{ mm}^4,$$

$$I_{yy2} = (1/18)18(3^3) = 27 \text{ mm}^4.$$

Part (3) $A_3 = \dfrac{\pi(9^2)}{2} = 127.23 \text{ mm}^2$,

$\mathbf{x}_3 = 9 \text{ mm}$,

$$\mathbf{y}_3 = 18 + \left(\frac{4(9)}{3\pi}\right) = 21.82 \text{ mm},$$

$$I_{xx3} = \left(\frac{1}{8}\right) \pi(9^4) - \left(\frac{4(9)}{3\pi}\right)^2 A_3 = 720.1 \text{ mm}^4,$$

$$I_{yy3} = \left(\frac{1}{8}\right) \pi(9^4) = 2576.5 \text{ mm}^4.$$

The composite area:

$$A = \sum_1^3 A_i = 397.23 \text{ mm}^2.$$

The area moment of inertia:

$$I_y = \mathbf{x}_1^2 A_1 + I_{yy1} - \mathbf{x}_2^2 A_2 - I_{yy2} + \mathbf{x}_3^2 A_3 + I_{yy3},$$

$$I_y = 4.347 \times 10^4 \text{ mm}^4,$$

$$k_y = \sqrt{\frac{I_y}{A}} = 10.461 \text{ mm}$$

Problem 8.65 Determine I_x and k_x.

Solution: Use the results of the solution to Problem 8.64.

$$I_X = \mathbf{y}_1^2 A_1 + I_{XX1} - \mathbf{y}_2^2 A_2 - I_{XX2} + \mathbf{y}_3^2 A_3 + I_{XX3},$$

$$I_x = 9.338 \times 10^4 \text{ mm}^4,$$

$$k_x = \sqrt{\frac{I_x}{A}} = 15.33 \text{ mm}$$

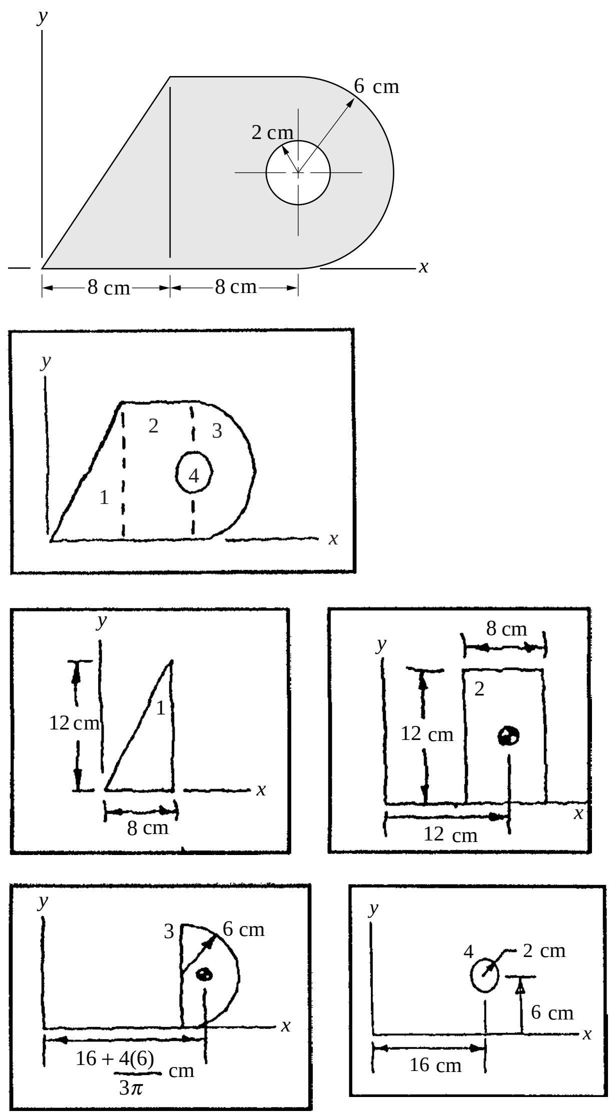
Problem 8.66 Determine I_{xy}.

Solution: Use the results of the solutions to Problems 8.63 and 8.64.

$$I_{xy} = \mathbf{x}_1\mathbf{y}_1 A_1 - \mathbf{x}_2\mathbf{y}_2 A_2 + \mathbf{x}_3\mathbf{y}_3 A_3$$

$$I_{xy} = 4.8313 \times 10^4 \text{ mm}^4$$

Problem 8.67 Determine I_y and k_y.



Solution: We divide the composite area into a triangle (1), rectangle (2), half-circle (3), and circular cutout (4):

Triangle:

$$(I_y)_1 = \tfrac{1}{4}(12)(8)^3 = 1536 \text{ cm}^4$$

Rectangle:

$$(I_y)_2 = \frac{1}{12}(12)(8)^3 + (12)^2(8)(12) = 14{,}336 \text{ cm}^4.$$

Half-Circle:

$$(I_y)_3 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(6)^4 + \left[16 + \frac{4(6)}{3\pi}\right]^2 \frac{1}{2}\pi(6)^2 = 19{,}593 \text{ cm}^4$$

Circular cutout:

$$(I_y)_4 = \tfrac{1}{4}\pi(2)^4 + (16)^2\pi(2)^2 = 3230 \text{ cm}^4.$$

Therefore

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4 = 3.224 \times 10^4 \text{ cm}^4.$$

The area is

$$A = A_1 + A_2 + A_3 - A_4$$

$$= \frac{1}{2}(12)(8) + (8)(12) + \frac{1}{2}\pi(6)^2 - \pi(2)^2 = 188 \text{ cm}^2,$$

so $k_y = \sqrt{\dfrac{I_y}{A}} = \sqrt{\dfrac{3.224 \times 10^4}{188}} = 13.1$ cm.

Problem 8.68 Determine J_O and k_O.

Solution: I_y is determined in the solution to Problem 8.67. We will determine I_x and use the relation $J_O = I_x + I_y$. Using the figures in the solution to Problem 8.67,

Triangle:

$$(I_x)_1 = \frac{1}{12}(8)(12)^3 = 1152 \text{ cm}^4.$$

Rectangle:

$$(I_x)_2 = \tfrac{1}{3}(8)(12)^3 = 4608 \text{ cm}^4.$$

Half Circle:

$$(I_x)_3 = \tfrac{1}{8}\pi(6)^4 + (6)^2\tfrac{1}{2}\pi(6)^2 = 2545 \text{ cm}^4.$$

Circular Cutout:

$$(I_x)_4 = \tfrac{1}{4}\pi(2)^4 + (6)^2\pi(2)^2 = 465 \text{ cm}^4.$$

Therefore

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4 = 7840 \text{ cm}^4.$$

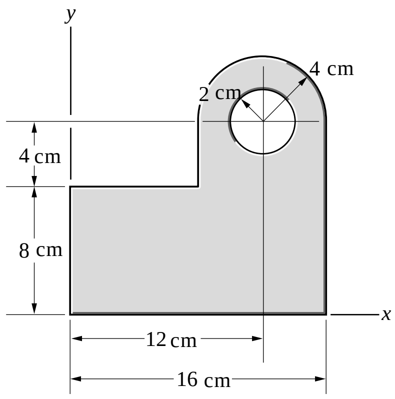
Using the solution of Problem 8.67,

$$J_O = I_x + I_y = 0.784 \times 10^4 + 3.224 \times 10^4 = 4.01 \times 10^4 \text{ cm}^4.$$

From the solution of Problem 8.67, $A = 188$ cm^2, so

$$R_0 = \sqrt{\frac{J_O}{A}} = \sqrt{\frac{4.01 \times 10^4}{188}} = 14.6 \text{ cm}.$$

Problem 8.69 Determine I_y and k_y.



Solution: Divide the area into four parts: Part (1) The rectangle 8 cm by 16 cm. Part (2): The rectangle 4 cm by 8 cm. Part (3) The semi-circle of radius 4 cm, and Part (4) The circle of radius 2 cm.

Part (1): $A_1 = 16(8) = 128\ \text{cm}^2$,

$\mathbf{x}_1 = 8\ \text{cm}$,

$\mathbf{y}_1 = 4\ \text{cm}$,

$$I_{xx1} = \left(\frac{1}{12}\right)16(8^3) = 682.67\ \text{cm}^4,$$

$$I_{yy1} = \left(\frac{1}{12}\right)8(16^3) = 2730.7\ \text{cm}^4.$$

Part (2): $A_2 = 4(8) = 32\ \text{cm}^2$,

$\mathbf{x}_2 = 12\ \text{cm}$,

$\mathbf{y}_2 = 10\ \text{cm}$,

$$I_{xx2} = \left(\frac{1}{12}\right)8(4^3) = 42.667\ \text{cm}^4,$$

$$I_{yy2} = \left(\frac{1}{12}\right)4(8^3) = 170.667\ \text{cm}^4.$$

Part (3): $A_3 = \dfrac{\pi(4^2)}{2} = 25.133\ \text{cm}^2$,

$\mathbf{x}_3 = 12\ \text{cm}$.

$$\mathbf{y}_3 = 12 + \left(\frac{4(4)}{3\pi}\right) = 13.698\ \text{cm}.$$

The area moments of inertia about the centroid of the semicircle are

$$I_{yy3} = \left(\frac{1}{8}\right)\pi(4^4) = 100.53\ \text{cm}^4,$$

$$I_{xx3} = \left(\frac{1}{8}\right)\pi(4^4) - \left(\frac{4(4)}{3\pi}\right)^2 A_3 = 28.1\ \text{cm}^4.$$

Check:

$I_{xx3} = 0.1098(R^4) = 28.1\ \text{cm}^4$.

check.

Part (4): $A_4 = \pi(2^2) = 12.566\ \text{cm}^2$,

$\mathbf{x}_4 = 12\ \text{cm}$,

$\mathbf{y}_4 = 12\ \text{cm}$,

$I_{xx4} = \left(\frac{1}{4}\right)\pi(2^4) = 12.566\ \text{cm}^4$,

$I_{yy4} = I_{xx4} = 12.566\ \text{cm}^4$.

The composite area:

$$A = \sum_1^3 A_i - A_4 = 172.566\ \text{cm}^2.$$

The area moment of inertia:

$$I_y = \mathbf{x}_1^2A_1 + I_{yy1} + \mathbf{x}_2^2A_2 + I_{yy2} + \mathbf{x}_3^2A_3 + I_{yy3} - \mathbf{x}_4^2A_4 - I_{yy4}$$

$I_y = 1.76 \times 10^4\ \text{cm}^4$,

$$k_y = \sqrt{\frac{I_y}{A}} = 10.1\ \text{cm}$$

Problem 8.70 Determine I_x and k_x.

Solution: Use the results in the solution to Problem 8.69.

$$I_x = \mathbf{y}_1^2A_1 + I_{xx1} + \mathbf{y}_2^2A_2 + I_{xx2} + \mathbf{y}_3^2A_3 + I_{xx3} - \mathbf{y}_4^2A_4 - I_{xx4}$$

$I_x = 8.89 \times 10^3\ \text{cm}^4$

$$k_x = \sqrt{\frac{I_x}{A}} = 7.18\ \text{cm}$$

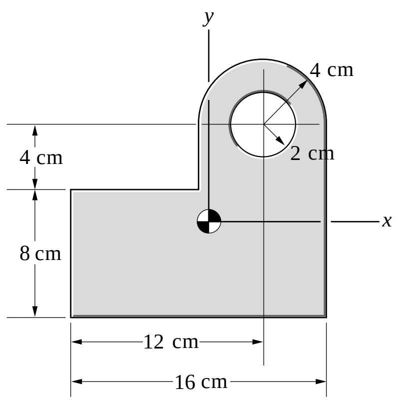
Problem 8.71 Determine I_{xy}.

Solution: Use the results in the solution to Problem 8.69.

$I_{xy} = \mathbf{x}_1\mathbf{y}_1A_1 + \mathbf{x}_2\mathbf{y}_2A_2 + \mathbf{x}_3\mathbf{y}_3A_3 - \mathbf{x}_4\mathbf{y}_4A_4,$

$I_{xy} = 1.0257 \times 10^4\ \text{cm}^4$

Problem 8.72 Determine I_y and k_y.



Solution: Use the results in the solutions to Problems 8.69 to 8.71. The centroid is

$$\mathbf{x} = \frac{\mathbf{x}_1A_1 + \mathbf{x}_2A_2 + \mathbf{x}_3A_3 - \mathbf{x}_4A_4}{A}$$

$$= \frac{1024 + 384 + 301.6 - 150.8}{172.567} = 9.033\ \text{cm},$$

from which

$I_{yc} = -\mathbf{x}^2A + I_y = -1.408 \times 10^4 + 1.7598 \times 10^4 = 3518.2\ \text{cm}^4$

$k_{yc} = \sqrt{\frac{I_{yc}}{A}} = 4.52\ \text{cm}$

Problem 8.73 Determine I_x and k_x.

Solution: Use the results in the solutions to Problems 8.69 to 8.71. The centroid is

$$\mathbf{y} = \frac{\mathbf{y}_1A_1 + \mathbf{y}_2A_2 + \mathbf{y}_3A_3 - \mathbf{y}_4A_4}{A} = 5.942\ \text{cm},$$

from which

$I_{xc} = -\mathbf{y}^2A + I_x = -6092.9 + 8894 = 2801\ \text{cm}^4$

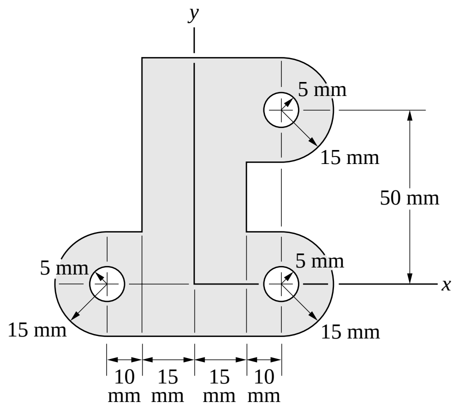
$k_{xc} = \sqrt{\frac{I_{xc}}{A}} = 4.03\ \text{cm}$

Problem 8.74 Determine I_{xy}.

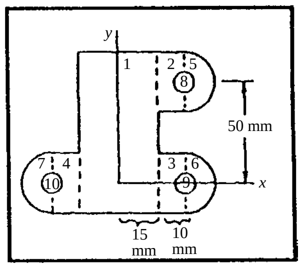
Solution: Use the results in the solutions to Problems 8.69–8.71.

$I_{xyc} = -\mathbf{xy}A + I_{xy} = -9.263 \times 10^3 + 1.0257 \times 10^4 = 994.5\ \text{cm}^4$

Problem 8.75 Determine I_y and k_y.



Solution: We divide the area into parts as shown:



$$(I_y)_1 = \frac{1}{12}(50+15+15)(30)^3 = 180{,}000 \text{ mm}^4$$

$$(I_y)_2 = (I_y)_3 = (I_y)_4 = \frac{1}{12}(30)(10)^3 + (20)^2(10)(30)$$

$$= 122{,}500 \text{ mm}^4$$

$$(I_y)_5 = (I_y)_6 = (I_y)_7 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(15)^4$$

$$+ \left[25 + \frac{4(15)}{3\pi}\right]^2 \frac{1}{2}\pi(15)^2 = 353{,}274 \text{ mm}^4$$

$$(I_y)_8 = (I_y)_9 = (I_y)_{10} = \frac{1}{4}\pi(5)^4 + (25)^2\pi(5)^2 = 49{,}578 \text{ mm}^4.$$

Therefore,

$$I_y = (I_y)_1 + 3(I_y)_2 + 3(I_y)_5 - 3(I_y)_8 = 1.46 \times 10^6 \text{ mm}^4.$$

The area is

$$A = A_1 + 3A_2 + 3A_5 - 3A_8$$

$$= (30)(80) + 3(10)(30) + 3\left(\frac{1}{2}\right)\pi(15)^2 - 3\pi(5)^2$$

$$= 4125 \text{ mm}^2$$

so $k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1.46 \times 10^6}{4125}} = 18.8$ mm

Problem 8.76 Determine J_O and k_O.

Solution: I_y is determined in the solution to Problem 8.75. We will determine I_x and use the relation $J_O = I_x + I_y$. Dividing the area as shown in the solution to Problem 8.75, we obtain

$$(I_x)_1 = \frac{1}{12}(30)(80)^3 + (25)^2(30)(80) = 2,780,000 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(10)(30)^3 + (50)^2(10)(30) = 772,500 \text{ mm}^4$$

$$(I_x)_3 = (I_x)_4 = \frac{1}{12}(10)(30)^3 = 22,500 \text{ mm}^4$$

$$(I_x)_5 = \frac{1}{8}\pi(15)^4 + (50)^2\frac{1}{2}\pi(15)^2 = 903{,}453 \text{ mm}^4$$

$$(I_x)_6 = (I_x)_7 = \frac{1}{8}\pi(15)^4 = 19{,}880 \text{ mm}^4,$$

$$(I_x)_8 = \frac{1}{4}\pi(5)^4 + \pi(5)^2(50)^2,$$

$$(I_x)_9 = (I_x)_{10} = \frac{1}{4}\pi(5)^4 = 491 \text{ mm}^4.$$

Therefore

$$I_x = (I_x)_1 + (I_x)_2 + 2(I_x)_3 + (I_x)_5 + 2(I_x)_6 - (I_x)_8 - 2(I_x)_9$$

$$= 4.34 \times 10^6 \text{ mm}^4$$

and $J_O = I_x + I_y = 5.80 \times 10^6$ mm^4.

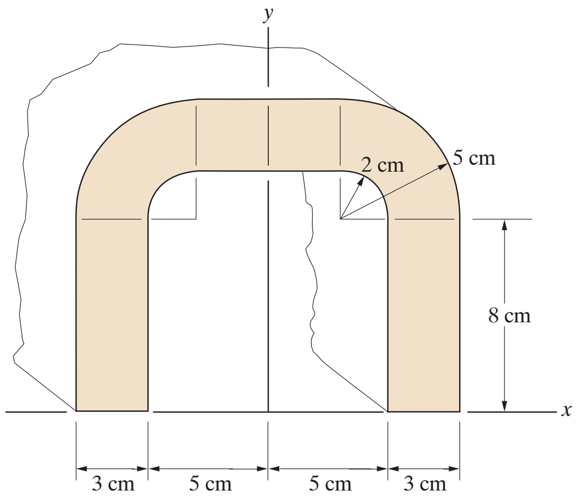
From the solution to Problem 8.75, $A = 4125$ mm^2

so $k_O = \sqrt{\frac{J_O}{A}}$

$$= \sqrt{\frac{5.80 \times 10^6}{4125}}$$

$$= 37.5 \text{ mm}.$$

Problem 8.77 Determine I_x and I_y for the beam's cross section.



Solution: Use the symmetry of the object

$$\frac{I_x}{2} = \frac{1}{3}(3\text{ cm})(8\text{ cm})^3 + \left[\frac{1}{12}(3\text{ cm})(3\text{ cm})^3 + (3\text{ cm})^2(11.5\text{ cm})^2\right]$$

$$+\left[\frac{\pi(5\text{ cm})^4}{16} - \frac{\pi(5\text{ cm})^2}{4}\left(\frac{4[5\text{ cm}]}{3\pi}\right)^2\right.$$

$$\left.+\frac{\pi(5\text{ cm})^2}{4}\left(8\text{ cm} + \frac{4[5\text{ cm}]}{3\pi}\right)^2\right]$$

$$-\left[\frac{\pi(2\text{ cm})^4}{16} - \frac{\pi(2\text{ cm})^2}{4}\left(\frac{4[2\text{ cm}]}{3\pi}\right)^2\right.$$

$$\left.+\frac{\pi(2\text{ cm})^2}{4}\left(8\text{ cm} + \frac{4[2\text{ cm}]}{3\pi}\right)^2\right]$$

Solving we find

$$\boxed{I_x = 7016\text{ cm}^4}$$

$$\frac{I_y}{2} = \frac{1}{3}(3\text{ cm})(3\text{ cm})^3 + \left[\frac{1}{12}(8\text{ cm})(3\text{ cm})^3 + (8\text{ cm})(3\text{ cm})(6.5\text{ cm})^2\right]$$

$$+\left[\frac{\pi(5\text{ cm})^4}{16} - \frac{\pi(5\text{ cm})^2}{4}\left(\frac{4[5\text{ cm}]}{3\pi}\right)^2\right.$$

$$\left.+\frac{\pi(5\text{ cm})^2}{4}\left(3\text{ cm} + \frac{4[5\text{ cm}]}{3\pi}\right)^2\right]$$

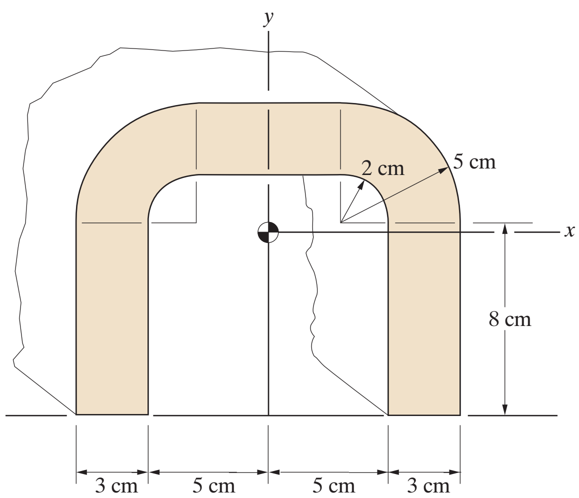
$$-\left[\frac{\pi(2\text{ cm})^4}{16} - \frac{\pi(2\text{ cm})^2}{4}\left(\frac{4[2\text{ cm}]}{3\pi}\right)^2\right.$$

$$\left.+\frac{\pi(2\text{ cm})^2}{4}\left(3\text{ cm} + \frac{4[2\text{ cm}]}{3\pi}\right)^2\right]$$

Solving we find

$$\boxed{I_y = 3122\text{ cm}^4}$$

Problem 8.78 Determine I_x and I_y for the beam's cross section.

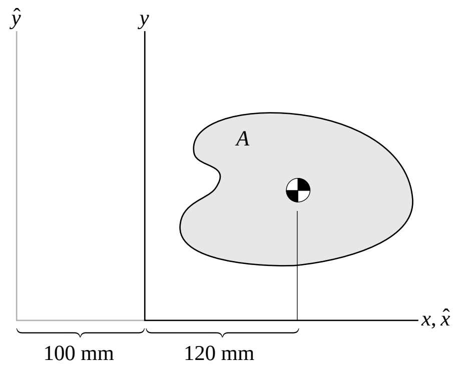


Solution: Use Solution 8.77 and 7.39. From Problem 7.39 we know that

$$\overline{y} = 7.48\text{ cm},\ A = 98.987\text{ cm}^2$$

$$\boxed{\begin{aligned} I_x &= 7016\text{ cm}^4 - A\overline{y}^2 = 1471\text{ cm}^4 \\ I_y &= 3122\text{ cm}^4 - A(0)^2 = 3122\text{ cm}^4 \end{aligned}}$$

Problem 8.79 The area $A = 2 \times 10^4$ mm^2. Its moment of inertia about the y axis is $I_y = 3.2 \times 10^8$ mm^4. Determine its moment of inertia about the $\hat{y}$ axis.



Solution: Use the parallel axis theorem. The moment of inertia about the centroid of the figure is

$$I_{yc} = -x^2A + I_y = -(120^2)(2 \times 10^4) + 3.2 \times 10^8$$

$$= 3.20 \times 10^7 \text{ mm}^4.$$

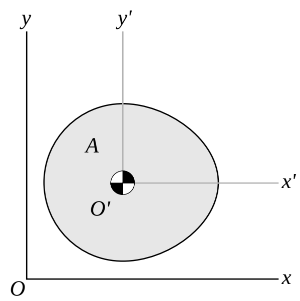
The moment of inertia about the $\hat{y}$ axis is

$$I_{\hat{y}} = \mathbf{x}^2A + I_{yc}$$

$$I_{\hat{y}} = (220^2)(2 \times 10^4) + 3.2 \times 10^7$$

$$= 1 \times 10^9 \text{ mm}^4$$

Problem 8.80 The area $A = 100$ mm^2 and it is *symmetric* about the x' axis. The moments of inertia $I_{x'} = 420$ mm^4, $I_{y'} = 580$ mm^4, $J_O = 11000$ mm^4, and $I_{xy} = 4800$ mm^4. What are I_x and I_y?



Solution: The basic relationships:

(1) $I_x = y^2A + I_{xc}$,
(2) $I_y = x^2A + I_{yc}$,
(3) $J_O = Ar^2 + J_c$,
(4) $J_O = I_x + I_y$,
(5) $J_c = I_{xc} + I_{yc}$, and
(6) $I_{xy} = Axy + I_{xyc}$,

where the subscript c applies to the primed axes, and the others to the unprimed axes. The x, y values are the displacement of the primed axes from the unprimed axes. The steps in the demonstration are:

(i) From symmetry about the x_c axis, the product of inertia $I_{xyc} = 0$.

(ii) From (3): $r^2 = \dfrac{J_O - J_c}{A} = 100$ mm^2, from which $r^2 = x^2 + y^2 = 100$ mm^2

(iii) From (6) and $I_{xyc} = 0$, $y = \dfrac{I_{xy}}{Ax}$, from which $x^2r^2 = x^4 + \left(\dfrac{I_{xy}}{A}\right)^2$. From which: $x^4 - 100x^2 + 2304 = 0$.

(iv) The roots: $x_1^2 = 64$, and $x_2^2 = 36$. The corresponding values of y are found from $y = \sqrt{r^2 - x^2}$ from which $(x_1, y_1) = (8, 6)$, and $(x_2, y_2) = (6, 8)$.

(v) Substitute these pairs to obtain the possible values of the area moments of inertia:

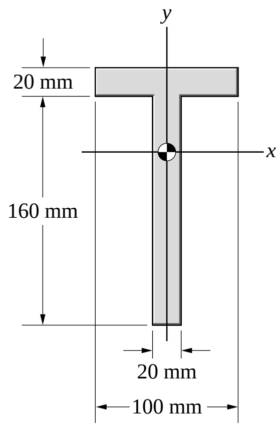
$$I_{x1} = Ay_1^2 + I_{xc} = 4020 \text{ mm}^4,$$

$$I_{y1} = Ax_1^2 + I_{yc} = 6980 \text{ mm}^4$$

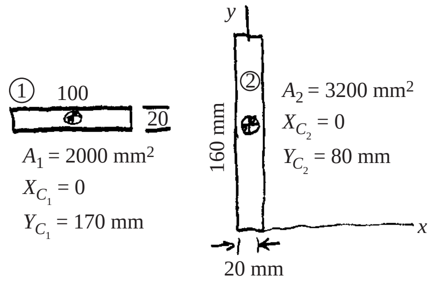
$$I_{x2} = Ay_2^2 + I_{xc} = 6820 \text{ mm}^4,$$

$$I_{y2} = Ax_2^2 + I_{yc} = 4180 \text{ mm}^4$$

Problem 8.81 Determine the moment of inertia of the beam cross section about the x axis. Compare your result with the moment of inertia of a solid square cross section of equal area. (See Example 8.5.)



Solution: We first need to find the location of the centroid of the composite. Break the area into two parts. Use X, Y coords.



For the composite

$$X_c = \frac{X_{c1}A_1 + X_{c2}A_2}{A_1 + A_2} = 0$$

$$Y_c = \frac{Y_{c1}A_1 + Y_{c2}A_2}{A_1 + A_2}$$

Substituting, we get

$X_c = 0$ mm

$Y_c = 114.6$ mm

We now find I_x for each part about its center and use the parallel axis theorem to find I_x about C.

Part (1): $b_1 = 100$ mm, $h_1 = 20$ mm

$$I_{x'1} = \frac{1}{12} b_1 h_1^3 = \frac{1}{12}(100)(20)^3 \text{ mm}^4$$

$I_{x'1} = 6.667 \times 10^4$ mm^4

$dy_1 = Y_{c1} - Y_c = 55.38$ mm

$I_{x1} = I_{x'1} + (dy_1)^2 (A_1)$

$I_{x1} = 6.20 \times 10^6$ mm^4

Part (2) $b_2 = 20$ mm, $h_2 = 160$ mm

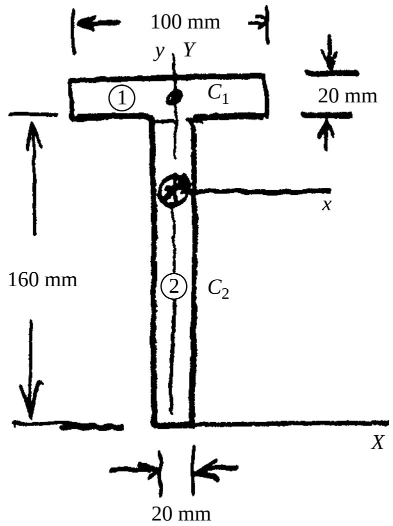
$$I_{x'2} = \frac{1}{12}(b_2)(h_2)^3 = \frac{1}{12}(20)(160)^3 \text{ mm}^4$$

$I_{x'2} = 6.827 \times 10^6$ mm^4

$dy_2 = Y_{c2} - Y_c = -34.61$ mm

$I_{x2} = I_{x'2} + (dy)^2 A_2$

$I_{x2} = 1.066 \times 10^7$ mm^4



Finally, $I_x = I_{x1} + I_{x2}$

$I_x = 1.686 \times 10^7$ mm^4

for our composite shape.

Now for the comparison. For the solid square with the same total area $A_1 + A_2 = 5200$ mm^2, we get a side of length

$l^2 = 5200$: $\quad l = 72.11$ mm

And for this solid section

$$I_{xSQ} = \frac{1}{12} bh^3 = \frac{1}{12} l^4$$

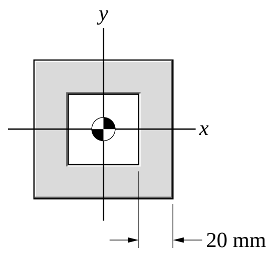
$I_{xSQ} = 2.253 \times 10^6$ mm^4

$$\text{Ratio} = I_x / I_{xSQ} = \frac{1.686 \times 10^7}{2.253 \times 10^6}$$

Ratio $= 7.48$

This matches the value in Example 8.5.

Problem 8.82 The area of the beam cross section is 5200 mm^2. Determine the moment of inertia of the beam cross section about the x axis. Compare your result with the moment of inertia of a solid square cross section of equal area. (See Example 8.5.)



Solution: Let the outside dimension be b mm, then the inside dimension is $b - 40$ mm. The cross section is $A = b^2 - (b - 40)^2 = 5200$ mm^2. Solve: $b = 85$ mm. Divide the beam cross section into two parts: the inner and outer squares. Part (1)

$$A_1 = 85^2 = 7225 \text{ mm}^2,$$

$$I_{xx1} = \left(\frac{1}{12}\right) 85(85^3) = 4.35 \times 10^6.$$

Part (2)

$$A_2 = 45^2 = 2025 \text{ mm}^2.$$

$$I_{xx2} = \left(\frac{1}{12}\right) 45(45^3) = 3.417 \times 10^5.$$

The composite moment of inertia about the centroid is

$$I_x = I_{xx1} - I_{xx2} = 4.008 \times 10^6 \text{ mm}^4.$$

For a square cross section of the same area, $h = \sqrt{5200} = 72.111$ mm.

The area moment of inertia is

$$I_{xb} = \left(\frac{1}{12}\right) 72.111(72.111^3) = 2.253 \times 10^6 \text{ in}^4.$$

The ratio:

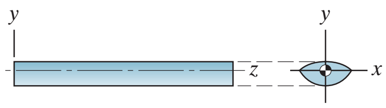
$$R = \frac{4.008 \times 10^6}{2.253 \times 10^6} = 1.7788 = 1.78$$

which confirms the value given in Example 8.5.

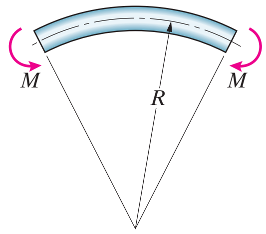
Problem 8.83 If the beam in Fig. a is subjected to couples of magnitude M about the x axis (Fig. b), the beam's longitudinal axis bends into a circular are whose radius R is given by

$$R = \frac{EI_x}{M},$$

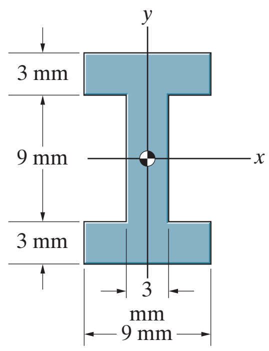
where I_x is the moment of inertia of the beam's cross section about the x axis. The value of the term E, which is called the *modulus of elasticity*, depends on the material of which the beam is constructed. Suppose that a beam with the cross section shown in Fig. c is subjected to couples of magnitude $M = 180$ N-m. As a result, the beam's axis bends into a circular arc with radius $R = 3$ m. What is the modulus of elasticity of the beam's material? (See Example 8.5.)



(a) Unloaded.



(b) Subjected to couples at the ends.



(c) Beam cross section.

Solution: The moment of inertia of the beam's cross section about the x axis is

$$I_x = \left\{\frac{1}{12}(3)(9)^3 + 2\left[\frac{1}{12}(9)(3)^3 + (6)^2(9)(3)\right]\right\} \text{ mm}^4$$

$$= 2170 \text{ mm}^4 = 2.17 \times 10^{-9} \text{ m}^4.$$

The modulus of elasticity is

$$E = \frac{RM}{I_x} = \frac{(3 \text{ m})(180 \text{ N-m})}{2.17 \times 10^{-9} \text{ m}^4} = 2.49 \times 10^{11} \text{ N/m}^2$$

$$\boxed{E = 2.49 \times 10^{11} \text{ N/m}^2.}$$

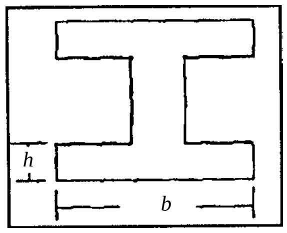
Problem 8.84 Suppose that you want to design a beam made of material whose density is 8000 kg/m^3. The beam is to be 4 m in length and have a mass of 320 kg. Design a cross section for the beam so that $I_x = 3 \times 10^{-5}$ m^4. (See Example 8.5.)

Solution: The strategy is to determine the cross sectional area, and then use the ratios given in Figure 8.14 to design a beam. The volume of the beam is $V = AL = 4A$ m^3. The mass of the beam is $m = V(8000) = 32000A = 320$ kg, from which $A = 0.01$ m^2. The moment of inertia for a beam of square cross section with this area is

$$I_{xxb} = \left(\frac{1}{12}\right)(0.1)(0.1^3) = 8.333 \times 10^{-6} \text{ m}^4.$$

The ratio is $R = \dfrac{3 \times 10^{-5}}{8.333 \times 10^{-6}} = 3.6.$

From Figure 8.6, this ratio suggests an I-beam of the form shown in the sketch. Choose an I-beam made up of three equal area rectangles, of dimensions b by hm in section. The moment of inertia about the centroid is $I_x = \mathbf{y}_1^2 A_1 + I_{xx1} + \mathbf{y}_2^2 A_2 + I_{xx2} + \mathbf{y}_3^2 A_3 + I_{xx3}$.

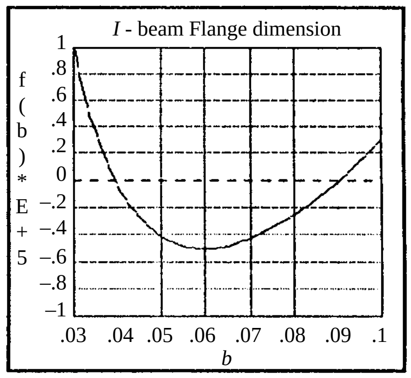


Since all areas are equal, $A_1 = A_2 = A_3 = bh$, and $\mathbf{y}_1 = \dfrac{b+h}{2}$, $\mathbf{y}_2 = 0$, and $\mathbf{y}_3 = -\mathbf{y}_1$, this reduces to

$$I_x = \left(\frac{1}{6}\right) bh^3 + 2\left(\frac{b+h}{2}\right)^2 hb + \left(\frac{1}{12}\right) hb^3.$$

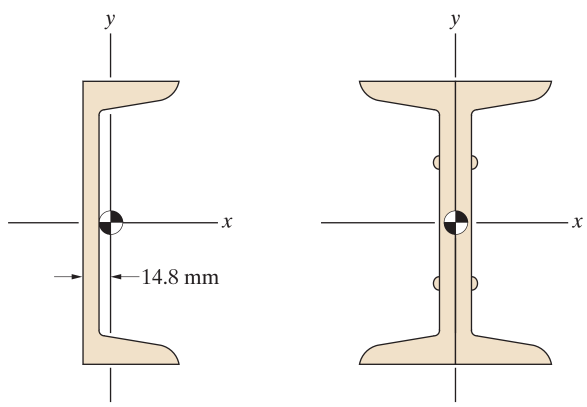
Note that $bh = \dfrac{A}{3}$, where A is the known total cross section area. These are two equations in two unknowns. Plot the function

$$f(b) = \left(\frac{1}{6}\right) bh^3 + 2\left(\frac{b+h}{2}\right)^2 bh + \left(\frac{1}{12}\right) hb^3 - I_x$$



subject to the condition that $hb = \dfrac{A}{3}$. The function was graphed using **TK Solver Plus.** The graph crosses the zero axis at approximately $b = 0.0395$ m. and $b = 0.09$ m. The lower value is an allowable value for h and the greater value corresponds to an allowable value of b. Thus the I beam design has the flange dimensions, $b = 90$ mm and $h = 39.5$ mm.

Problem 8.85 The area in Fig. (a) is a C230×30 American Standard Channel beam cross section. Its cross sectional area is $A = 3790$ mm^2 and its moments of inertia about the x and y axes are $I_x = 25.3 \times 10^6$ mm^4 and $I_y = 1 \times 10^6$ mm^4. Suppose that two beams with C230×30 cross sections are riveted together to obtain a composite beam with the cross section shown in Fig. (b). What are the moments of inertia about the x and y axes of the composite beam?

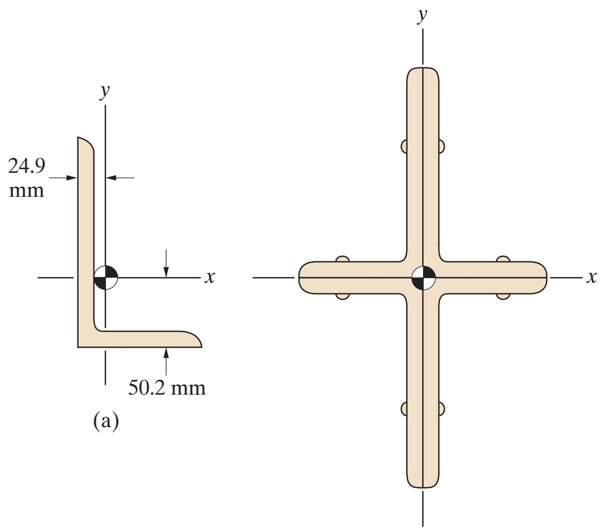


(a) (b)

Solution:

$$\boxed{\begin{aligned} I_x &= 2(25.3 \times 10^6 \text{ mm}^4) = 50.6 \times 10^6 \text{ mm}^4 \\ I_y &= 2(10^6 \text{ mm}^4 + [3790 \text{ mm}^2][14.8 \text{ mm}]^2) = 3.66 \times 10^6 \text{ mm}^4 \end{aligned}}$$

Problem 8.86 The area in Fig. (a) is an L152×102×12.7 Angle beam cross section. Its cross sectional area is $A = 3060\ \text{mm}^2$ and its moments of inertia about the x and y axes are $I_x = 7.24 \times 10^6\ \text{mm}^4$ and $I_y = 2.61 \times 10^6\ \text{mm}^4$. Suppose that four beams with L152×102×12.7 cross sections are riveted together to obtain a composite beam with the cross section shown in Fig. (b). What are the moments of inertia about the x and y axes of the composite beam?



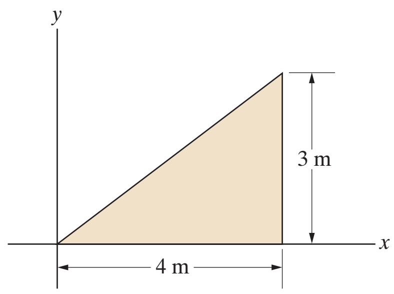
(a)

(b)

Solution:

$$\boxed{\begin{aligned} I_x &= 4(7.24 \times 10^6\ \text{mm}^4 + [3060\ \text{mm}^2][50.2\ \text{mm}]^2) \\ &= 59.8 \times 10^6\ \text{mm}^4 \\ I_y &= 4(2.61 \times 10^6\ \text{mm}^4 + [3060\ \text{mm}^2][24.9\ \text{mm}]^2) \\ &= 18.0 \times 10^6\ \text{mm}^4 \end{aligned}}$$

Problem 8.87 In Active Example 8.6, suppose that the vertical 3-m dimension of the triangular area is increased to 4 m. Determine a set of principal axes and the corresponding principal moments of inertia.



Solution: From Appendix B, the moments and products of inertia of the area are

$$I_x = \frac{1}{12}(4\ \text{m})(4\ \text{m})^3 = 21.3\ \text{m}^4,$$

$$I_y = \frac{1}{4}(4\ \text{m})^3(4\ \text{m}) = 64\ \text{m}^4,$$

$$I_{xy} = \frac{1}{8}(4\ \text{m})^2(4\ \text{m})^2 = 32\ \text{m}^4.$$

From Eq. (8.26),

$$\tan(2\theta_p) = \frac{2I_{xy}}{I_y - I_x} = \frac{2(32)}{64 - 21.3} = 1.50 \Rightarrow \theta_p = 28.2^\circ.$$

From Eqs. (8.23) and (8.24), the principal moments of inertia are

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta_p - I_{xy}\sin 2\theta_p$$

$$= \left(\frac{21.3 + 64}{2}\right) + \left(\frac{21.3 - 64}{2}\right)\cos(2[28.2^\circ]) - (32)\sin(2[28.2^\circ])$$

$$= 4.21\ \text{m}^4$$

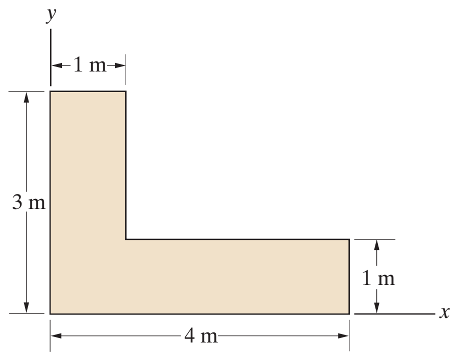
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta_p + I_{xy}\sin 2\theta_p$$

$$= \left(\frac{21.3 + 64}{2}\right) - \left(\frac{21.3 - 64}{2}\right)\cos(2[28.2^\circ]) + (32)\sin(2[28.2^\circ])$$

$$= 81.8\ \text{m}^4$$

$$\boxed{\begin{aligned} &\theta_p = 28.2^\circ,\ \text{principal moments of inertia are} \\ &\quad 4.21\ \text{m}^4,\ 81.1\ \text{m}^4 \end{aligned}}$$

Problem 8.88 In Example 8.7, suppose that the area is reoriented as shown. Determine the moments of inertia $I_{x'}$, $I_{y'}$ and $I_{x'y'}$ if $\theta = 30^o$.



Solution: Based on Example 8.7, the moments and product of inertia of the reoriented area are

$$I_x = 10 \text{ m}^4, I_y = 22 \text{ m}^4, \ I_{xy} = 6 \text{ m}^4 .$$

Applying Eqs. (8.23)–(8.25),

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{10+22}{2} + \frac{10-22}{2}\cos 60^\circ - 6\sin 60^\circ = 7.80 \text{ m}^4,$$

$$I_{y'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

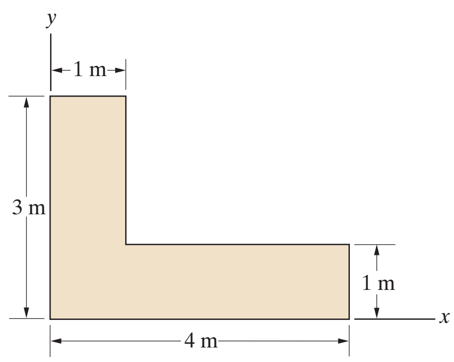
$$= \frac{10+22}{2} - \frac{10-22}{2}\cos 60^\circ + 6\sin 60^\circ = 24.2 \text{ m}^4,$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{12-22}{2}\sin 60^\circ + 6\cos 60^\circ = -2.20 \text{ m}^4$$

$$\boxed{I_{x'} = 7.80 \text{ m}^4, I_{y'} = 24.2 \text{ m}^4, I_{x'y'} = -2.20 \text{ m}^4.}$$

Problem 8.89 In Example 8.7, suppose that the area is reoriented as shown. Determine a set of principal axes and the corresponding principal moments of inertia. Based on the result of Example 8.7, can you predict a value of θ_p without using Eq. (8.26)?



Solution: Based on Example 8.7, the moments and product of inertia of the reoriented area are

$$I_x = 10 \text{ m}^4, I_y = 22 \text{ m}^4, I_{xy} = 6 \text{ m}^4 .$$

From Eq. (8.26), $\tan 2\theta_p = \dfrac{2I_{xy}}{I_x - I_y} = \dfrac{2(6)}{22-10} = 1 \Rightarrow \theta_p = 22.5^\circ$

This value could have been anticipated from Example 8.7 by reorienting the axes.

Substituting the angle into Eqs. (8.23) and (8.24), the principal moments of inertia are

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta_p - I_{xy}\sin 2\theta_p$$

$$= \frac{10+22}{2} + \frac{10-22}{2}\cos 45^\circ - 6\sin 45^\circ = 7.51 \text{ m}^4,$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta_p + I_{xy}\sin 2\theta_p$$

$$= \frac{10+22}{2} - \frac{10-22}{2}\cos 45^\circ + 6\sin 45^\circ = 24.5 \text{ m}^4,$$

$$\boxed{\theta_p = 22.5^\circ, \text{ principal moments of inertia are } 7.51 \text{ m}^4, 24.5 \text{ m}^4.}$$

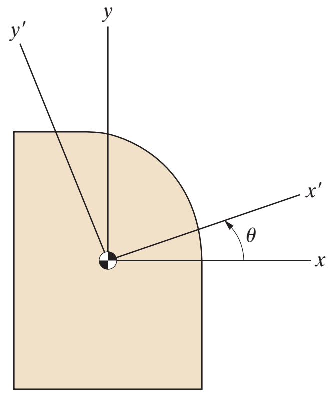
Problem 8.90 The moment of inertia of the area are

$I_x = 1.26 \times 10^6\ \text{cm}^4$,

$I_y = 6.55 \times 10^5\ \text{cm}^4$,

$I_{xy} = -1.02 \times 10^5\ \text{cm}^4$

Determine the moments of inertia of the area $I_{x'}$, $I_{y'}$ and $I_{x'y'}$ if $\theta = 30°$.



Solution:

Applying Eqs. (8.23)–(8.25),

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta\cdot$$

$$= \left[\frac{1.26 + 0.655}{2} + \frac{1.26 - 0.655}{2}\cos 60^\circ - (-0.102)\sin 60^\circ\right] \times 10^6\ \text{cm}^4$$

$$= 1.20 \times 10^6\ \text{cm}^4$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \left[\frac{1.26 + 0.655}{2} - \frac{1.26 - 0.655}{2}\cos 60^\circ + (-0.102)\sin 60^\circ\right] \times 10^6\ \text{cm}^4$$

$$= 7.18 \times 10^5\ \text{cm}^4$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left[\frac{1.26 - 0.655}{2}\sin 60^\circ + (-0.102)\cos 60^\circ\right] \times 10^6\ \text{cm}^4$$

$$= 2.11 \times 10^5\ \text{cm}^4$$

$$\boxed{I_{x'} = 1.20 \times 10^6\ \text{cm}^4,\ I_{y'} = 7.18 \times 10^5\ \text{cm}^4,\ I_{x'y'} = 2.11 \times 10^5\ \text{cm}^4.}$$

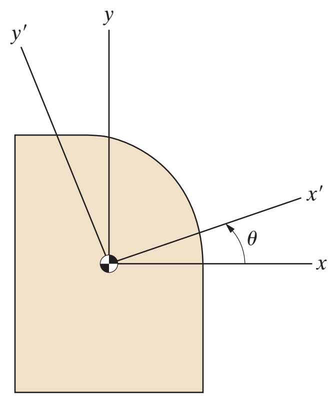
Problem 8.91 The moment of inertia of the area are

$I_x = 1.26 \times 10^6\ \text{cm}^4$,

$I_y = 6.55 \times 10^5\ \text{cm}^4$,

$I_{xy} = -1.02 \times 10^5\ \text{cm}^4$

Determine a set of principal axes and the corresponding principal moments of inertia.



Solution: From Eq. (8.26),

$$\tan 2\theta_p = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-.102)}{0.655 - 1.26} = 0.337$$

$$\Rightarrow \theta_p = 9.32^\circ$$

Substituting this angle into Eqs. (8.23) and (8.24), the principal moments of inertia are

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta_p - I_{xy}\sin 2\theta_p$$

$$= \left[\frac{1.26 + 0.655}{2} + \frac{1.26 - 0.655}{2}\cos 18.63^\circ - (-0.102)\sin 18.63^\circ\right]$$

$$\times 10^6\ \text{cm}^4 = 1.28 \times 10^6\ \text{cm}^4$$

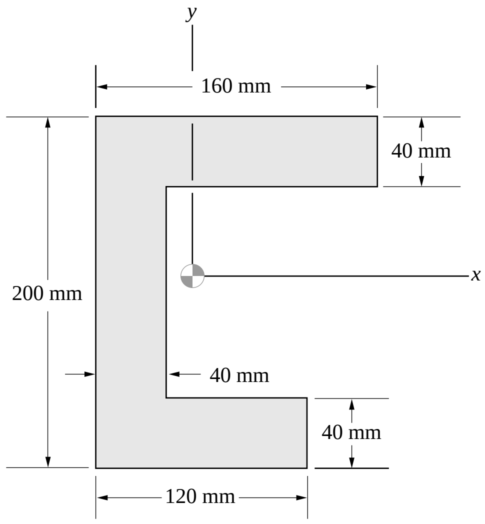
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta_p + I_{xy}\sin 2\theta_p$$

$$= \left[\frac{1.26 + 0.655}{2} - \frac{1.26 - 0.655}{2}\cos 18.63^\circ + (-0.102)\sin 18.63^\circ\right]$$

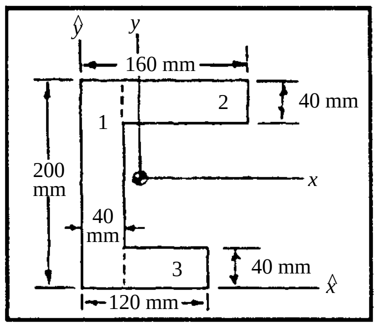
$$\times 10^6\ \text{cm}^4 = 6.38 \times 10^5\ \text{cm}^4$$

$\theta_p = 9.32^\circ$, principal moments of inertia are $1.28 \times 10^6\ \text{cm}^4$, $6.38 \times 10^5\ \text{cm}^4$.

Problem 8.92* Determine a set of principal axes and the corresponding principal moments of inertia.



Solution: We divide the area into 3 rectangles as shown: In terms of the $\hat{x}$, $\hat{y}$ coordinate system, the position of the centroid is

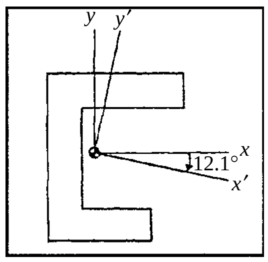


$$\hat{\mathbf{x}} = \frac{\hat{\mathbf{x}}_1 A_1 + \hat{\mathbf{x}}_2 A_1 + \hat{\mathbf{x}}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(20)(40)(200) + (100)(120)(40) + (80)(80)(40)}{(40)(200) + (120)(40) + (80)(40)} = 56 \text{ mm},$$

$$\hat{\mathbf{y}} = \frac{\hat{\mathbf{y}}_1 A_1 + \hat{\mathbf{y}}_2 A_1 + \hat{\mathbf{y}}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(100)(40)(200) + (180)(120)(40) + (20)(80)(40)}{(40)(200) + (120)(40) + (80)(40)} = 108 \text{ mm}.$$



The moments and products of inertia in terms of the $\hat{x}$, $\hat{y}$ system are

$$\widehat{I}_x = (\widehat{I}_x)_1 + (\widehat{I}_x)_2 + (\widehat{I}_x)_3$$

$$= \frac{1}{3}(40)(200)^3 + \frac{1}{12}(120)(40)^3 + (180)^2(120)(40)$$

$$+ \frac{1}{3}(80)(40)^3 = 26.5 \times 10^7 \text{ mm}^4,$$

$$\widehat{I}_y = (\widehat{I}_y)_1 + (\widehat{I}_y)_2 + (\widehat{I}_y)_3$$

$$= \frac{1}{3}(200)(40)^3 + \frac{1}{12}(40)(120)^3 + (100)^2(120)(40)$$

$$+ \frac{1}{12}(40)(80)^3 + (80)^2(80)(40) = 8.02 \times 10^7 \text{ mm}^4,$$

$$\widehat{I}_{xy} = (\widehat{I}_{xy})_1 + (\widehat{I}_{xy})_2 + (\widehat{I}_{xy})_3$$

$$= (20)(100)(40)(200) + (100)(180)(40)(120)$$

$$+ (20)(80)(40)(80) = 10.75 \times 10^7 \text{ mm}.$$

The moments and product of inertia in terms of the $\hat{x}$, $\hat{y}$ system are

$$I_x = \widehat{I}_x - (\hat{\mathbf{y}})^2 A = 77.91 \times 10^6 \text{ mm}^4,$$

$$I_y = \widehat{I}_y - (\hat{\mathbf{x}})^2 A = 30.04 \times 10^6 \text{ mm}^4,$$

$$I_{xy} = \widehat{I}_{xy} - \hat{\mathbf{x}}\hat{\mathbf{y}} A = 10.75 \times 10^6 \text{ mm}^4,$$

from Equation (8.26),

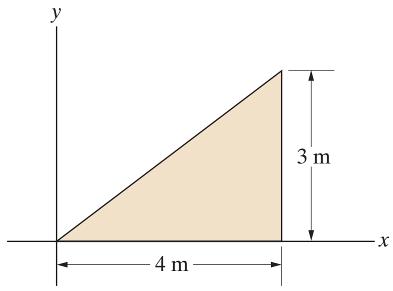
$$\tan 2\theta_p = \frac{2I_{xy}}{I_y - I_x} = \frac{2(10.75 \times 10^6)}{(30.04 \times 10^6) - (77.91 \times 10^6)},$$

we obtain $\theta p = -12.1^\circ$. We can orient the principal axes as shown: Substituting the values of I_x, I_y and I_{xy} into Equations (8.23) and (8.24) and setting $\theta = -12.1^\circ$, we obtain

$$I_{x^1} = 80.2 \times 10^6 \text{ mm}^4$$

$$I_{y^1} = 27.7 \times 10^6 \text{ mm}^4.$$

Problem 8.93 Solve Problem 8.87 by using Mohr's Circle.

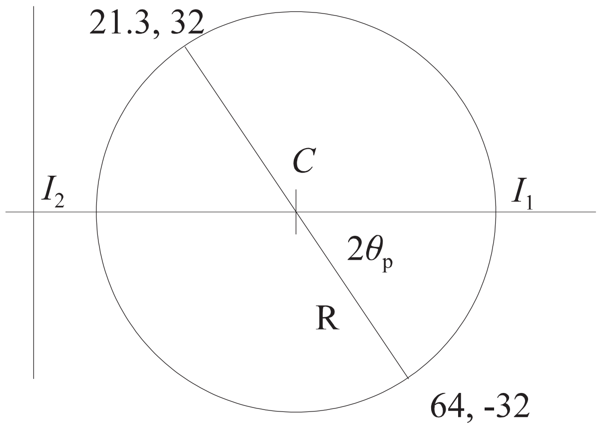


Solution: The vertical 3-m dimension is increased to 4 m. From Problem 8.87, the moments and product of inertia for the unrotated system are

$$I_x = \frac{1}{12}(4\text{ m})(4\text{ m})^3 = 21.3\text{ m}^4,$$

$$I_y = \frac{1}{4}(4\text{ m})^3(4\text{ m}) = 64\text{ m}^4,$$

$$I_{xy} = \frac{1}{8}(4\text{ m})^2(4\text{ m})^2 = 32\text{ m}^4.$$



Mohr's circle (shown) has a center and radius given by

$$C = \frac{21.3+64}{2} = 42.7\text{ m}^4$$

$$R = \sqrt{\left(\frac{21.3-64}{2}\right)^2 + (32)^2} = 38.5\text{ m}^4$$

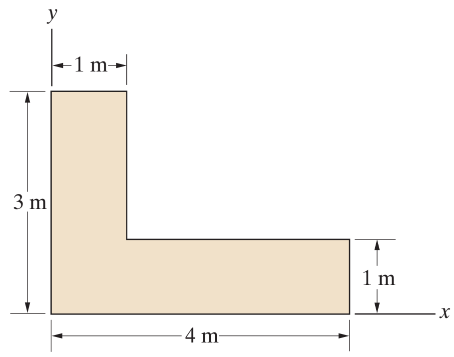
The angle and principal moments are now

$$\tan(2\theta_p) = \frac{32}{64-42.7} \Rightarrow \theta_p = 28.2^\circ,$$

$$I_1 = C + R = 81.1\text{ m}^4, I_2 = C - R = 4.21\text{ m}^4.$$

$$\boxed{\theta_p = 28.2^\circ,\ \text{principal moments of inertia are } 4.21\text{ m}^4, 81.1\text{ m}^4}$$

Problem 8.94 Solve Problem 8.88 by using Mohr's Circle.



Solution: Based on Example 8.7, the moments and product of inertia of the reoriented area are

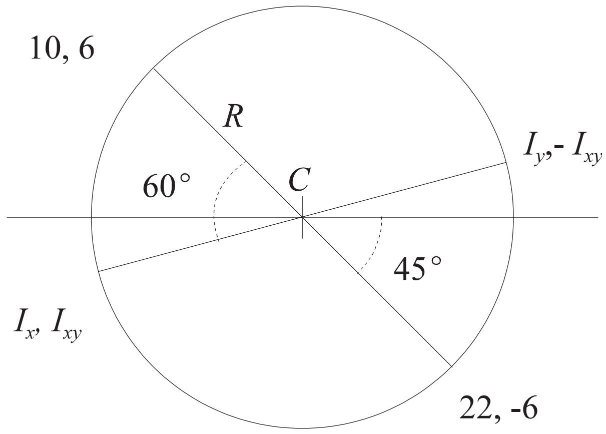
$$I_x = 10 \text{ m}^4, I_y = 22 \text{ m}^4, I_{xy} = 6 \text{ m}^4.$$

For Mohr's circle we have the center, radius, and angle

$$C = \frac{10 + 22}{2} = 16 \text{ m}^4,$$

$$R = \sqrt{\left(\frac{22 - 10}{2}\right)^2 + (6)^2} = 8.49 \text{ m}^4,$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{6}{22 - 16}\right) = 22.5^\circ$$



Now we can calculate the new inertias

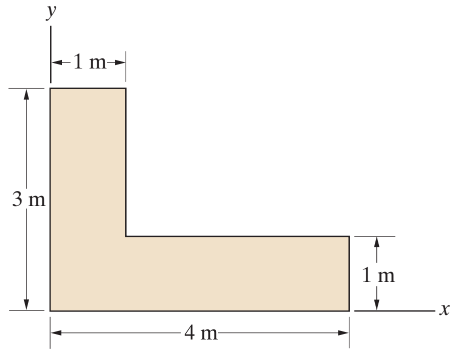
$$I_x = C - R\cos 15^\circ = 7.80 \text{ m}^4$$

$$I_y = C + R\cos 15^\circ = 24.2 \text{ m}^4$$

$$I_{xy} = -R\sin 15^\circ = -2.20 \text{ m}^4$$

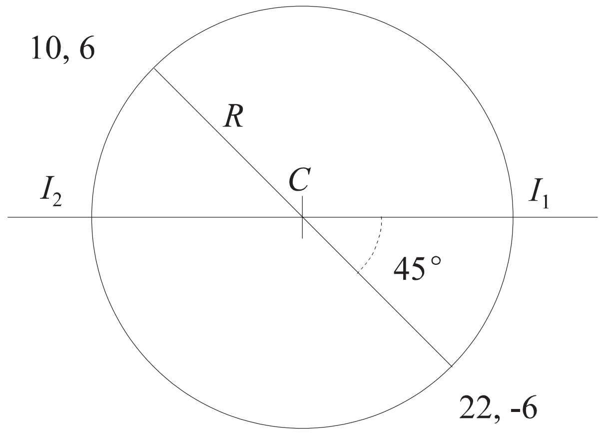
$$\boxed{I_{x'} = 7.80 \text{ m}^4, I_{y'} = 24.2 \text{ m}^4, I_{x'y'} = -2.20 \text{ m}^4.}$$

Problem 8.95 Solve Problem 8.89 by using Mohr's Circle.



Solution: Based on Example 8.7, the moments and product of inertia of the reoriented area are

$$I_x = 10 \text{ m}^4, I_y = 22 \text{ m}^4, I_{xy} = 6 \text{ m}^4.$$



For Mohr's circle we have the center, radius, and angle

$$C = \frac{10+22}{2} = 16 \text{ m}^4,$$

$$R = \sqrt{\left(\frac{22-10}{2}\right)^2 + (6)^2} = 8.49 \text{ m}^4,$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{6}{22-16}\right) = 22.5^\circ$$

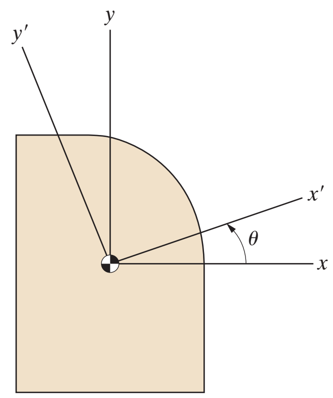
Now we can calculate the principal moments of inertias

$$I_1 = C + R = 24.5 \text{ m}^4$$

$$I_2 = C - R = 7.51 \text{ m}^4$$

$\boxed{\theta_p = 22.5^\circ, \text{ principal moments of inertia are } 7.51 \text{ m}^4, 24.5 \text{ m}^4.}$

Problem 8.96 Solve Problem 8.90 by using Mohr's Circle.



Solution: For Mohr's circle we have the center, radius, and angle

$$C = \left(\frac{12.6 + 6.55}{2}\right) \times 10^5 = 9.58 \times 10^5 \text{ in}^4,$$

$$R = \sqrt{\left(\frac{12.6 - 6.55}{2}\right)^2 + (-1.02)^2} \times 10^5 = 3.19 \text{ in}^4,$$

$$\theta_\text{p} = \frac{1}{2}\tan^{-1}\left(\frac{1.02}{12.6 - 9.58}\right) = 9.32^\circ$$

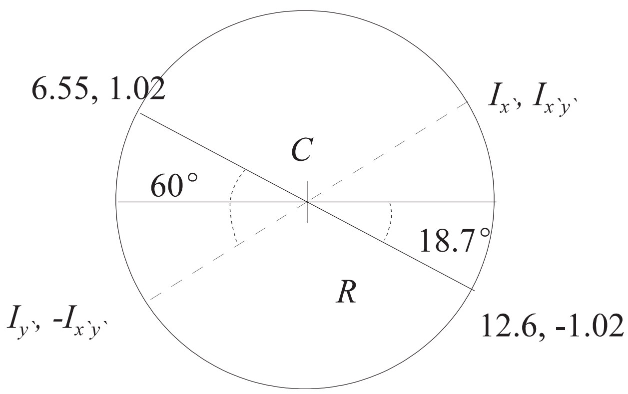
Now we can calculate the new inertias

$$I_{x'} = C + R\cos 22.7^\circ = 12.0 \times 10^5 \text{ in}^4,$$

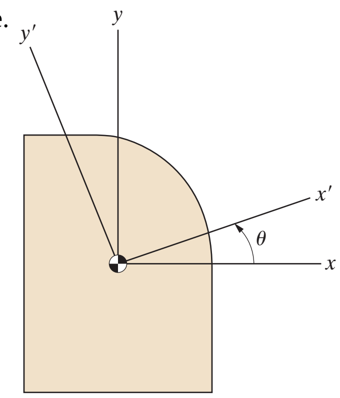
$$I_{y'} = C - R\cos 22.7^\circ = 7.18 \times 10^5 \text{ in}^4,$$

$$I_{x'y'} = R\sin 22.7^\circ = 2.11 \times 10^5 \text{ in}^4.$$

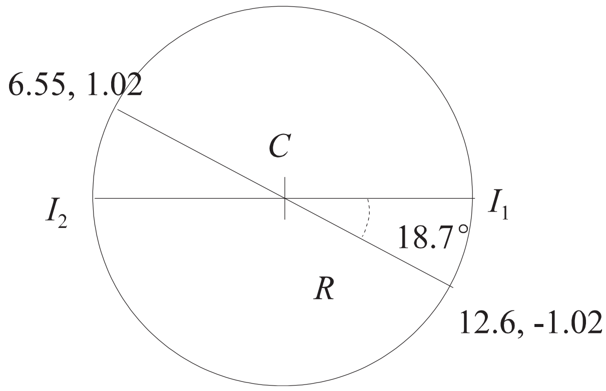
$$\boxed{I_{x'} = 1.20 \times 10^6 \text{ in}^4, I_{y'} = 7.18 \times 10^5 \text{ in}^4, I_{x'y'} = 2.11 \times 10^5 \text{ in}^4.}$$



Problem 8.97 Solve Problem 8.91 by using Mohr's Circle.



Solution: For Mohr's circle we have the center, radius, and angle



$$C = \left(\frac{12.6 + 6.55}{2}\right) \times 10^5 = 9.58 \times 10^5 \text{ in}^4,$$

$$R = \sqrt{\left(\frac{12.6 - 6.55}{2}\right)^2 + (-1.02)^2} \times 10^5 = 3.19 \text{ in}^4,$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{1.02}{12.6 - 9.58}\right) = 9.32^\circ$$

Now we can calculate the principal inertias

$$I_1 = C + R = 12.8 \times 10^5 \text{ in}^4,$$

$$I_{y'} = C - R = 6.38 \times 10^5 \text{ in}^4,$$

$$\boxed{\begin{array}{l}\theta_p = 9.32^\circ, \text{ principal moments of inertia are} \\ \quad 1.28 \times 10^6 \text{ in}^4, 6.38 \times 10^5 \text{ in}^4.\end{array}}$$

Problem 8.98* Solve Problem 8.92 by using Mohr's circle.

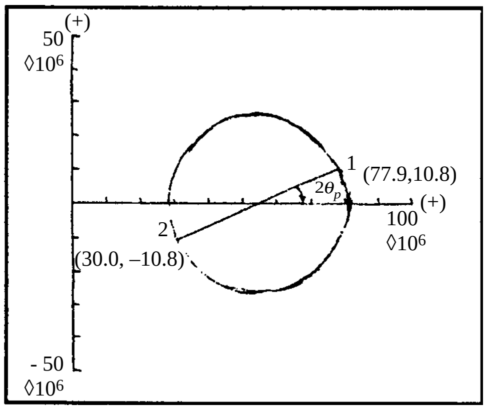
Solution: The moments and product of inertia are derived in terms of the xy coordinate system in the solution of Problem 8.92:

$$I_x = 77.91 \times 10^6 \text{ mm}^4$$

$$I_y = 30.04 \times 10^6 \text{ mm}^4$$

$$I_{xy} = 10.75 \times 10^6 \text{ mm}^4.$$

The Mohr's circle is: Measuring the $2\theta p$, angle we estimate that $\theta p = -12°$, and the principle moments of inertia are approximately 81×10^6 mm^4 and 28×10^6 mm^4 the orientation of the principal axes is shown in the solution of Problem 8.92.



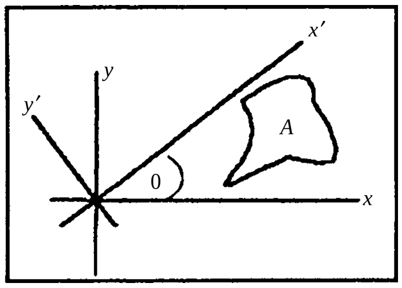
Problem 8.99 Derive Eq. (8.22) for the product of inertia by using the same procedure we used to derive Eqs. (8.20) and (8.21).

Solution: Suppose that the area moments of inertia of the area A are known in the coordinate system (x, y),

$$I_x = \int_A y^2 \, dA,$$

$$I_y = \int_A x^2 \, dA,$$

and $I_{xy} = \int_A xyA.$



The objective is to find the product of inertia in the new coordinate system (x', y') in terms of the known moments of inertia. The new (x', y') system is formed from the old (x, y) system by rotation about the origin through a counterclockwise angle θ.

By definition,

$$I_{x'y'} = \int_A x'y' \, dA.$$

From geometry,

$$x' = x\cos\theta + y\sin\theta,$$

and $y' = -x\sin\theta + y\cos\theta.$

The product is

$$x'y' = xy\cos^2\theta - xy\sin^2\theta + y^2\cos\theta\sin\theta - x^2\cos\theta\sin\theta.$$

Substitute into the definition:

$$I_{x'y'} = (\cos^2\theta - \sin^2\theta)\int_A xy\,dA$$

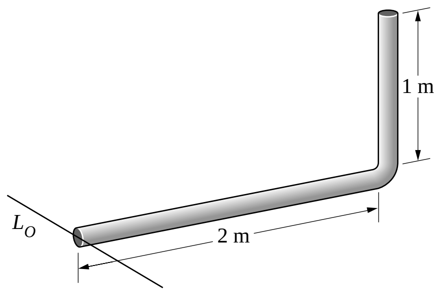
$$+ (\cos\theta\sin\theta)\left(\int_A y^2\,dA - \int_A x^2\,dA\right),$$

from which

$$I_{x'y'} = (\cos^2\theta - \sin^2\theta)I_{xy} + (I_x - I_y)\sin\theta\cos\theta,$$

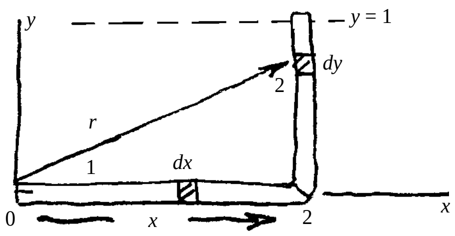
which is the expression required.

Problem 8.100 The axis L_O is perpendicular to both segments of the L-shaped slender bar. The mass of the bar is 6 kg and the material is homogeneous. Use the method described in Example 8.10 to determine is moment of inertia about L_O.



Solution: Use Example 8.10 as a model for this solution.

Introduce the coordinate system shown and divide the bar into two parts as shown



$$(I_0)_1 = \int r^2\,dm = \int_0^2 \rho A x^2\,dx = \rho A \frac{x^3}{3}\Big|_0^2$$

$$(I_0)_1 = \frac{8}{3}\rho A$$

However $m_1 = \rho A l_1 = (2\rho A)$.

Since part 1 is 2/3 of the length, its mass is $2/3(6 \text{ kg}) = 4$ kg. Part 2 has mass 2 kg.

For part 2, $dm = \rho A\,dy$ and

$$r = \sqrt{2^2 + y^2}$$

$$(I_0)_2 = \int_{m_2} r^2\,dm = \int_0^1 \rho A(2^2 + y^2)\,dy$$

$$(I_0)_2 = \rho A(4y + \frac{y^3}{3})\Big|_0^1 = \rho A \frac{13}{3}$$

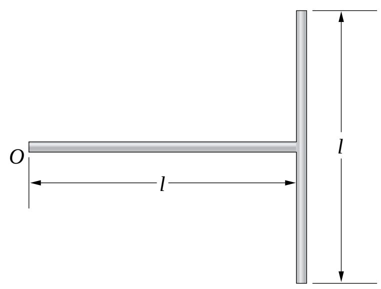
$$I_{0_{\text{TOTAL}}} = \frac{13}{3}\rho A + \frac{8}{3}\rho A = \frac{21}{3}\rho A$$

$$(I_0)_{\text{TOTAL}} = 7\rho A$$

The total mass $= 3\rho A = 6$ kg

$$I_{0_{\text{TOTAL}}} = \frac{7}{6}(6 \text{ kg}) \cdot m^2 = 7 \text{ kg m}^2$$

Problem 8.101 Two homogenous slender bars, each of mass m and length l, are welded together to form the T-shaped object. Use integration to determine the moment of inertia of the object about the axis through point O that is perpendicular to the bars.



Solution: Divide the object into two pieces, each corresponding to a slender bar of mass m; the first parallel to the y axis, the second to the x axis. By definition

$$I = \int_0^l r^2\, dm + \int_m r^2\, dm.$$

For the first bar, the differential mass is $dm = \rho A\, dr$. Assume that the second bar is very slender, so that the mass is concentrated at a distance l from O. Thus $dm = \rho A\, dx$, where x lies between the limits

$$-\frac{l}{2} \le x \le \frac{l}{2}.$$

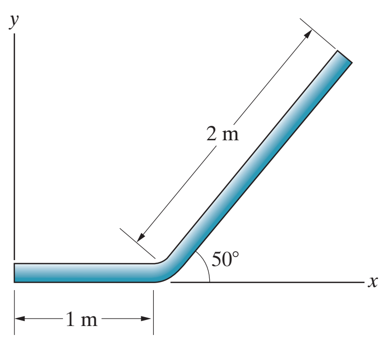
The distance to a differential dx is $r = \sqrt{l^2 + x^2}$. Thus the definition becomes

$$I = \rho A \int_0^l r^2\, dr + \rho A \int_{-\frac{l}{2}}^{\frac{l}{2}} (l^2 + x^2)\, dx\ I$$

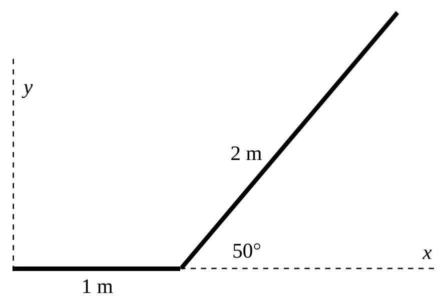
$$= \rho A \left[\frac{r^3}{3}\right]_0^l + \rho A \left[l^2 x + \frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$= ml^2 \left(\frac{1}{3} + 1 + \frac{1}{12}\right) = \frac{17}{12} ml^2$$

Problem 8.102 The slender bar lies in the $x-y$ plane. Its mass is 6 kg and the material is homogeneous. Use integration to determine its moment of inertia about the z axis.



Solution: The density is $\rho = \dfrac{6\text{ kg}}{3\text{ m}} = 2\text{ kg/m}$



$$I_z = \int_0^{1\text{ m}} \rho x^2 dx$$

$$+ \int_0^{2\text{ m}} \rho[(1\text{ m} + s\cos 50^\circ)^2 + (s \sin 50^\circ)^2] ds$$

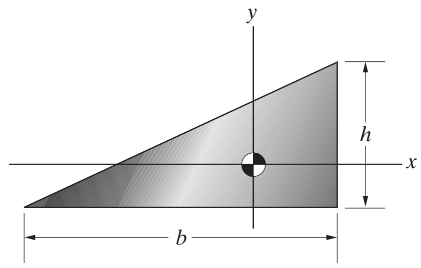
$$\boxed{I_z = 15.14\text{ kg m}^2}$$

Problem 8.103 Use integration to determine the moment of inertia of the slender bar in Problem 8.102 about the y axis.

Solution: See solution for 8.102

$$\boxed{I_y = \int_0^{1\text{ m}} \rho x^2 dx + \int_0^{2\text{ m}} \rho(1\text{ m} + s\cos 50^\circ)^2 ds = 12.01\text{ kg m}^2}$$

Problem 8.104 The homogeneous thin plate has mass $m = 12$ kg and dimensions $b = 2$ m and $h = 1$ m. Use the procedure described in Active Example 8.9 to determine the moments of inertia of the plate about the x and y axes.



Solution: From Appendix B, the moments of inertia about the x and y axes are

$$I_x = \frac{1}{36}bh^3, I_y = \frac{1}{36}hb^3.$$

Therefore the moments of inertia of the plate about the x and y axes are

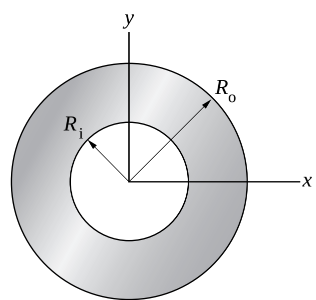
$$I_{x\text{axis}} = \frac{m}{A}I_x - \frac{m}{\frac{1}{2}bh}\left(\frac{1}{36}b^3\right) = \frac{1}{18}mh^2 = \frac{1}{8}(12\text{ kg})(1\text{ m})^2 = 0.667\text{ kg-m}^2$$

$$I_{y\text{axis}} = \frac{m}{A}I_y = \frac{m}{\frac{1}{2}bh}\left(\frac{1}{36}hb^3\right) = \frac{1}{18}mb^2 = \frac{1}{18}(12\text{ kg})(2\text{ m})^2 = 2.67\text{ kg-m}^2$$

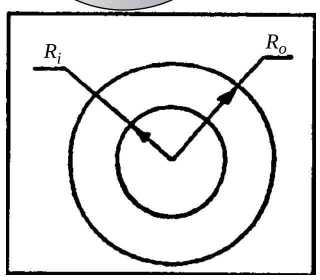
$$\boxed{I_{x\text{axis}} = 0.667\text{ kg-m}^2, I_{y\text{axis}} = 2.67\text{ kg-m}^2.}$$

Problem 8.105 The homogenous thin plate is of uniform thickness and mass m.

(a) Determine its moments of inertia about the x and z axes.
(b) Let $R_i = 0$, and compare your results with the values given in Appendix C for a thin circular plate.



Solution:



(a) The area moments of inertia for a circular area are $I_x = I_y = \frac{\pi R^4}{4}$. For the plate with a circular cutout, $I_x = \frac{\pi}{4}(R_o^4 - R_i^4)$. The area mass density is $\frac{m}{A}$, thus for the plate with a circular cut, $\frac{m}{A} = \frac{m}{\pi(R_o^2 - R_i^2)}$, from which the moments of inertia

$$I_{x\text{ axis}} = \frac{m(R_o^4 - R_i^4)}{4(R_o^2 - R_i^2)} = \frac{m}{4}(R_o^2 + R_i^2)$$

$$I_{z\text{ axis}} = 2I_{x\text{ axis}} = \frac{m}{2}(R_o^2 + R_i^2).$$

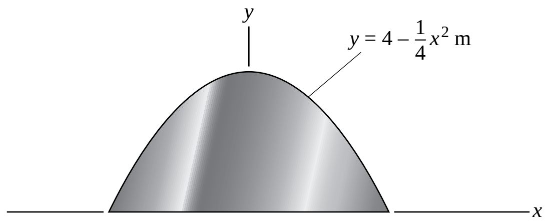
(b) Let $R_i = 0$, to obtain

$$I_{x\text{ axis}} = \frac{m}{4}R_o^2,$$

$$I_{z\text{ axis}} = \frac{m}{2}R_o^2,$$

which agrees with table entries.

Problem 8.106 The homogenous thin plate is of uniform thickness and weighs 200 N. Determine its moment of inertia about the y axis.



Solution:

$y = 4 - \frac{1}{4}x^2$ m

The plate's area is

$$A = \int_{-4}^{4} \left(4 - \frac{1}{4}x^2\right) dx = 21.3 \text{ m}^2.$$

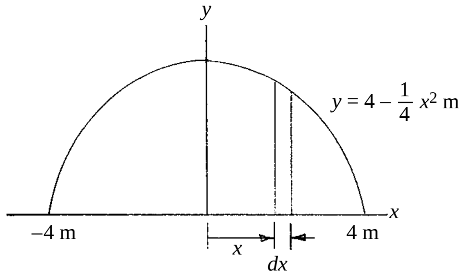
The plate's density *per unit area* is

$\delta = (200/9.81)/21.3 = 0.9558$ kg/m^2.

The moment of inertia about the y axis is

$$I_{y \text{ axis}} = \int_{-4}^{4} x^2 \delta \left(4 - \frac{1}{4}x^2\right) dx$$

$$= 65.3 \text{ kg-m}^2.$$



Problem 8.107 Determine the moment of inertia of the plate in Problem 8.106 about the x axis.

Solution: See the solution of Problem 8.106. The mass of the strip element is

$$m_{(\text{strip})} = \delta \left(4 - \frac{1}{4}x^2\right) dx.$$

The moment of inertia of the strip about the x axis is

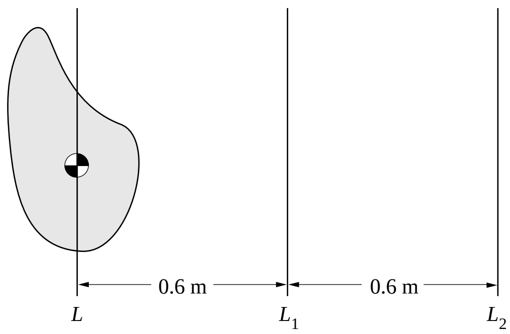
$$I_{(\text{strip})} = \frac{1}{3} m_{(\text{strip})} \left(4 - \frac{1}{4}x^2\right)^2$$

$$= \frac{1}{3}\delta \left(4 - \frac{1}{4}x^2\right)^3 dx,$$

so the moment of inertia of the plate about the x axis is

$$I_{(x \text{ axis})} = \int_{-4}^{4} \frac{1}{3}\delta \left(4 - \frac{1}{4}x^2\right)^3 dx = 74.6 \text{ kg-m}^2.$$

Problem 8.108 The mass of the object is 10 kg. Its moment of inertia about L_1 is 10 kg-m^2. What is its moment of inertia about L_2? (The three axes lie in the same plane.)

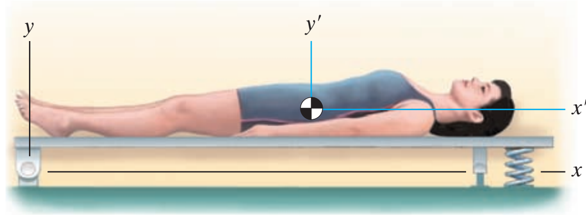


Solution: The strategy is to use the data to find the moment of inertia about L, from which the moment of inertia about L_2 can be determined.

$$I_L = -(0.6)^2(10) + 10 = 6.4 \text{ m}^2,$$

from which $I_{L_2} = (1.2)^2(10) + 6.4 = 20.8$ m^2

Problem 8.109 An engineer gathering data for the design of a maneuvering unit determines that the astronaut's center of mass is at $x = 1.01$ m, $y = 0.16$ m and that her moment of inertia about the z axis is 105.6 kg-m^2. Her mass is 81.6 kg. What is her moment of inertia about the z' axis through her center of mass?



Solution: The distance d from the z axis to the z' axis is

$$d = \sqrt{(1.01)^2 + (0.16)^2}$$

$$= 1.0226 \text{ m.}$$

From the parallel-axis theorem,

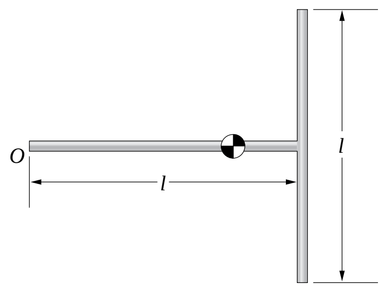
$$I_{(z \text{ axis})} = I_{(z' \text{ axis})} + d^2\, m:$$

$$105.6 = I_{(z' \text{ axis})} + (1.0226)^2(81.6).$$

Solving, we obtain

$$I_{(z' \text{ axis})} = 20.27 \text{ kg-m}^2.$$

Problem 8.110 Two homogenous slender bars, each of mass m and length l, are welded together to form the T-shaped object. Use the parallel axis theorem to determine the moment of inertia of the object about the axis through point O that is perpendicular to the bars.



Solution: Divide the object into two pieces, each corresponding to a bar of mass m. By definition

$$I = \int_0^l r^2\, dm.$$

For the first bar, the differential mass is $dm = \rho A\, dr$, from which the moment of inertia about one end is

$$I_1 = \rho A \int_0^l r^2\, dr = \rho A \left[\frac{r^3}{3}\right]_0^l = \frac{ml^2}{3}.$$

For the second bar

$$I_2 = \rho A \int_{-\frac{l}{2}}^{\frac{l}{2}} r^2\, dr = \rho A \left[\frac{r^3}{3}\right]_{-\frac{l}{2}}^{\frac{l}{2}} = \frac{ml^2}{12}$$

is the moment of inertia about the center of the bar. From the parallel axis theorem, the moment of inertia about O is

$$I_o = \frac{ml^2}{3} + l^2 m + \frac{ml^2}{12} = \frac{17}{12} ml^2.$$

Problem 8.111 Use the parallel-axis theorem to determine the moment of inertia of the T-shaped object in Problem 8.110 about the axis through the center of mass of the object that is perpendicular to the two bars. (See Active Example 8.11.)

Solution: The location of the center of mass of the object is $\mathbf{x} = \dfrac{m\left(\frac{l}{2}\right) + lm}{2m} = \dfrac{3}{4}l$. Use the results of Problem 8.110 for the moment of inertia of a bar about its center. For the first bar,

$$I_1 = \left(\frac{l}{4}\right)^2 m + \frac{ml^2}{12} = \frac{7}{48}ml^2.$$

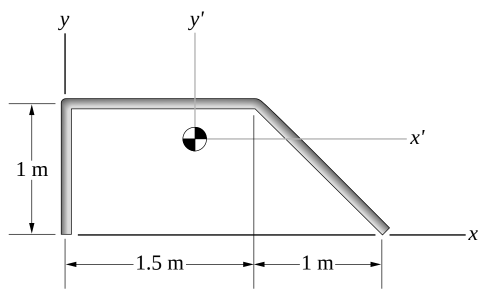
For the second bar,

$$I_2 = \left(\frac{l}{4}\right)^2 m + \frac{ml^2}{12} = \frac{7}{48}ml^2.$$

The composite:

$$I_c = I_1 + I_2 = \frac{7}{24}ml^2$$

Problem 8.112 The mass of the homogenous slender bar is 20 kg. Determine its moment of inertia about the z axis.



Solution: Divide the object into three segments. Part (1) is the 1 m bar on the left, Part (2) is the 1.5 m horizontal segment, and Part (3) is the segment on the far right. The mass density *per unit length* is

$$\rho = \frac{m}{L} = \frac{20}{(1 + 1.5 + \sqrt{2})} = 5.11 \text{ kg/m}.$$

The moments of inertia about the centers of mass and the distances to the centers of mass from the z axis are:

Part (1) $\quad I_1 = \rho\left(\dfrac{l_1^3}{12}\right) = m_1\dfrac{l_1^2}{12} = 0.426 \text{ kg-m}^2,$

$$m_1 = 5.11 \text{ kg},$$

$$d_1 = 0.5 \text{ m},$$

Part (2), $\quad I_2 = \rho\dfrac{l_2^3}{12} = m_2\dfrac{l_2^2}{12} = 1.437 \text{ kg- m}^2,$

$$m_2 = 7.66 \text{ kg},$$

$$d_2 = \sqrt{0.75^2 + 1^2} = 1.25 \text{ m}$$

Part (3) $\quad I_3 = \rho\dfrac{l_3^3}{12} = m_3\dfrac{(\sqrt{2})^2}{12} = 1.204 \text{ kg-m}^2,$

$$m_3 = 7.23 \text{ kg},$$

$$d_3 = \sqrt{2^2 + 0.5^2} = 2.062 \text{ m}.$$

The composite:

$$I = d_1^2m_1 + I_1 + d_2^2m_2 + I_2 + d_3^2m_3 + I_3 = 47.02 \text{ kg-m}^2$$

Problem 8.113 Determine the moment of inertia of the bar in Problem 8.112 about the z' axis through its center of mass.

Solution: The center of mass:

$$\mathbf{x} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{20}$$

$$= \frac{0 + 0.75(7.66) + 2(7.23)}{20} = 1.01 \text{ m}.$$

$$\mathbf{y} = \frac{0.5 m_1 + 1 m_2 + 0.5 m_3}{20}$$

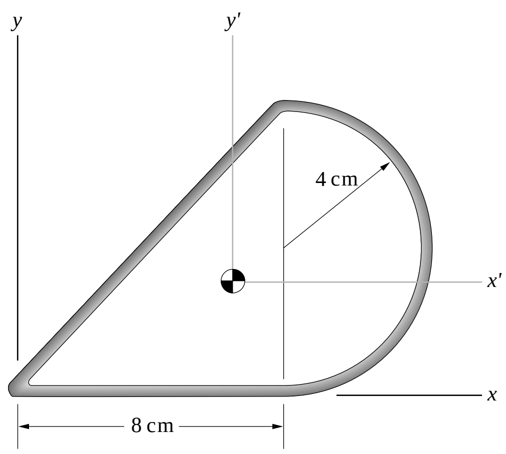
$$= \frac{0.5(5.11) + 1(7.66) + 0.5(7.23)}{20} = 0.692 \text{ m}.$$

The distance from the z axis to the center of mass is $d = \sqrt{\mathbf{x}^2 + \mathbf{y}^2} = 1.224$ m. The moment of inertia about the center o mass:

$$I_c = -d^2(20) + I_o$$

$$= 17.1 \text{ kg-m}^2$$

Problem 8.114 The homogeneous slender bar weighs 5 N. Determine its moment of inertia about the z axis.



Solution: The Bar's mass is $m = 5/9.81$ kg. Its length is

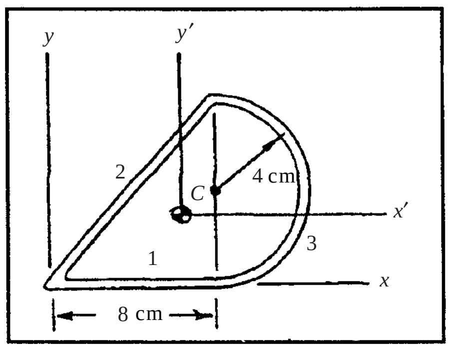
$$L = L_1 + L_2 + L_3 = 8 + \sqrt{8^2 + 8^2} + \pi(4) = 31.9 \text{ cm}.$$

The masses of the parts are therefore,

$$m_1 = \frac{L_1}{L} m = \left(\frac{8}{31.9}\right)\left(\frac{5}{9.81}\right) = 0.1279 \text{ kg}.$$

$$m_2 = \frac{L_2}{L} m = \left(\frac{\sqrt{2(64)}}{31.9}\right)\left(\frac{5}{9.81}\right) = 0.1809 \text{ kg},$$

$$m_3 = \frac{L_3}{L} m = \left(\frac{4\pi}{31.9}\right)\left(\frac{5}{9.81}\right) = 0.2009 \text{ kg}.$$



The center of mass of part 3 is located to the right of its center C a distance $2R/\pi = 2(4)/\pi = 2.55$ cm. The moment of inertia of part 3 about C is

$$\int_{m_3} r^2 \, dm = m_3 r^2 = (0.2009)(4)^2 = 3.2145 \text{ kg-cm}^2.$$

The moment of inertia of part 3 about the center of mass of part 3 is therefore

$$I_3 = 3.2145 - m_3(2.55)^2 = 1.9117 \text{ kg-cm}^2.$$

The moment of inertia of the bar about the z axis is

$$I_{(z \text{ axis})} = \tfrac{1}{3} m_1 L_1^2 + \tfrac{1}{3} m_2 L_2^2 + I_3 + m_3[(8 + 2.55)^2 + (4)^2]$$

$$= 37.9 \text{ kg-cm}^2 = 3.79 \times 10^{-3} \text{ kg-m}^2.$$

Problem 8.115 Determine the moment of inertia of the bar in Problem 8.114 about the z' axis through its center of mass.

Solution: In the solution of Problem 8.114, it is shown that the moment of inertia of the bar about the z axis is $I_{(z\ \text{axis})} = 37.9$ kg-cm^2. The x and y coordinates of the center of mass coincide with the centroid of the axis:

$$\mathbf{x} = \frac{\mathbf{x}_1 L_1 + \mathbf{x}_2 L_2 + \mathbf{x}_3 L_3}{L_1 + L_2 + L_3}$$

$$= \frac{(4)(8) + (4)\sqrt{8^2+8^2} + \left[8 + \dfrac{2(4)}{\pi}\right]\pi(4)}{8 + \sqrt{8^2+8^2} + \pi(4)} = 6.58 \text{ cm},$$

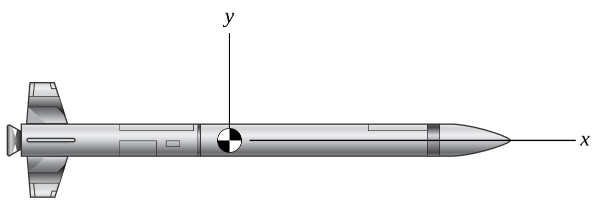
$$\mathbf{y} = \frac{\mathbf{y}_1 L_1 + \mathbf{y}_2 L_2 + \mathbf{y}_3 L_3}{L_1 + L_2 + L_3}$$

$$= \frac{0 + (4)\sqrt{8^2+8^2} + (4)\pi(4)}{8 + \sqrt{8^2+8^2} + \pi(4)} = 3.00 \text{ cm}.$$

The moment of inertia about the z axis is

$$I_{(z'\ \text{axis})} = I_{(z\ \text{axis})} - (\mathbf{x}^2 + \mathbf{y}^2)\left(\frac{5}{32.2}\right) = 11.3 \text{ kg-cm}^2.$$

Problem 8.116 The rocket is used for atmospheric research. Its weight and its moment of inertia about the z axis through its center of mass (including its fuel) are 50 kN and 13,770 kg-m^2, respectively. The rocket's fuel weighs 30 kN, its center of mass is located at $x = -0.9$ m, $y = 0$, $z = 0$, and the moment of inertia of the fuel about the axis through the fuel's center of mass parallel to z is 2970 kg-m^2. When the fuel is exhausted, what is the rocket's moment of inertia about the axis through its new center of mass parallel to z?



Solution: Denote the moment of inertia of the empty rocket as I_E about a center of mass x_E, and the moment of inertia of the fuel as I_F about a mass center x_F. Using the parallel axis theorem, the moment of inertia of the filled rocket is

$$I_R = I_E + x_E^2 m_E + I_F + x_F^2 m_F,$$

about a mass center at the origin ($x_R = 0$). Solve:

$$I_E = I_R - x_E^2 m_E - I_F - x_F^2 m_F.$$

The objective is to determine values for the terms on the right from the data given. Since the filled rocket has a mass center at the origin, the mass center of the empty rocket is found from $0 = m_E x_E + m_F x_F$, from which

$$x_E = -\left(\frac{m_F}{m_E}\right) x_F.$$

Using a value of $g = 9.81$ m/s^2,

$$m_F = \frac{W_F}{g} = \frac{6000}{9.81} = 3058.1 \text{ kg},$$

$$m_E = \frac{(W_R - W_F)}{g} = \frac{50{,}000 - 30{,}000}{9.81} = 2038.7 \text{ kg}.$$

From which

$$x_E = -\left(\frac{3058.1}{2038.4}\right)(-0.9) = 1.35 \text{ m}$$

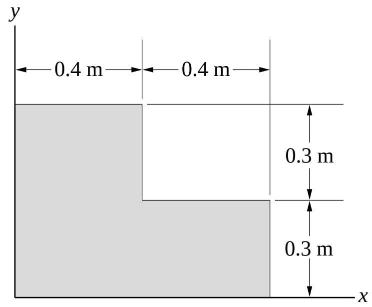
is the new location of the center of mass. Substitute:

$$I_E = I_R - x_E^2 m_E - I_F - x_F^2 m_F$$

$$= 13770 - 3715.6 - 2970 - 2477.1$$

$$= 4607.3 \text{ kg-m}^2$$

Problem 8.117 The mass of the homogenous thin plate is 36 kg. Determine its moment of inertia about the x axis.



Solution: Divide the plate into two areas: the rectangle 0.4 m by 0.6 m on the left, and the rectangle 0.4 m by 0.3 m on the right. The mass density is $\rho = \frac{m}{A}$. The area is

$$A = (0.4)(0.6) + (0.4)(0.3) = 0.36 \text{ m}^2,$$

from which

$$\rho = \frac{36}{0.36} = 100 \text{ kg/m}^2.$$

The moment of inertia about the x axis is

$$I_{x \text{ axis}} = \rho\left(\tfrac{1}{3}\right)(0.4)(0.6^3) + \rho\left(\tfrac{1}{3}\right)(0.4)(0.3)^3 = 3.24 \text{ kg-m}^2$$

Problem 8.118 Determine the moment of inertia of the plate in Problem 8.117 about the z axis.

Solution: The basic relation to use is

$$I_{z \text{ axis}} = I_{x \text{ axis}} + I_{y \text{ axis}}.$$

The value of $I_{x \text{ axis}}$ is given in the solution of Problem 8.117. The moment of inertia about the y axis using the same divisions as in Problem 8.117 and the parallel axis theorem is

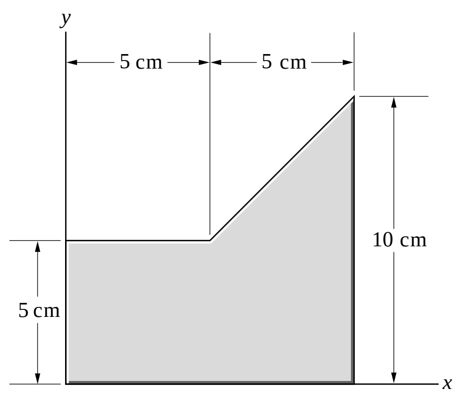
$$I_{y \text{ axis}} = \rho\left(\frac{1}{3}\right)(0.6)(0.4)^3 + \rho\left(\frac{1}{12}\right)(0.3)(0.4)^3$$

$$+ (0.6)^2\rho(0.3)(0.4) = 5.76 \text{ kg-m}^2,$$

from which

$$I_{z \text{ axis}} = I_{x \text{ axis}} + I_{y \text{ axis}} = 3.24 + 5.76 = 9 \text{ kg-m}^2.$$

Problem 8.119 The homogenous thin plate weighs 10 N. Determine its moment of inertia about the x axis.



Solution: Divide the area into two parts: the lower rectangle 5 cm by 10 cm and the upper triangle 5 cm base and 5 cm altitude. The mass density is $\rho = \frac{W}{gA}$. The area is

$$A = 5(10) + \left(\tfrac{1}{2}\right)5(5) = 62.5 \text{ cm}^2.$$

Using $g = 9.81 \text{ m/s}^2$, the mass density is

$$\rho = \frac{W}{gA} = 0.01631 \text{ kg/cm}^2.$$

Using the parallel axis theorem, the moment of inertia about the x axis is

$$I_{x \text{ axis}} = \rho\left(\frac{1}{3}\right)(10)(5)^3 + \rho\left(\frac{1}{36}\right)(5)(5^3)$$

$$+ \rho\left(5 + \frac{5}{3}\right)^2\left(\frac{1}{2}\right)(5)(5) = 16.14 \text{ kg-cm}^2$$

$$I_{x \text{ axis}} = 1.614 \times 10^{-3} \text{ kg-m}^2.$$

Problem 8.120 Determine the moment of inertia of the plate in Problem 8.119 about the y axis.

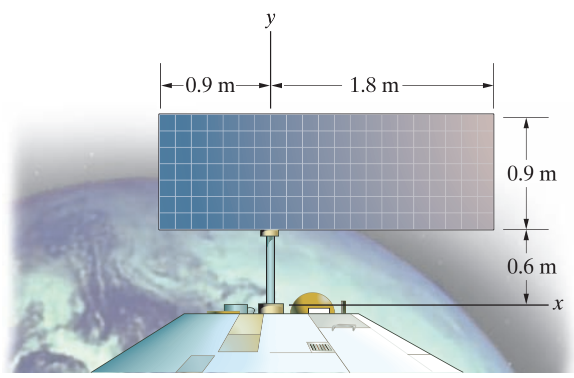
Solution: Use the results of the solution in Problem 8.119 for the area and the mass density.

$$I_{y \text{ axis}} = \rho \left(\frac{1}{3}\right) 5(10^3) + \rho \left(\frac{1}{36}\right) 5(5^3)$$

$$+ \rho \left(5 + \frac{10}{3}\right)^2 \left(\frac{1}{2}\right) 5(5)$$

$$= 41.62 \text{ kg-cm}^2 = 4.162 \times 10^{-3} \text{ kg-m}^2$$

Problem 8.121 The thermal radiator (used to eliminate excess heat from a satellite) can be modeled as a homogenous, thin rectangular plate. Its mass is 80 kg. Determine its moment of inertia about the x, y, and z axes.



Solution: The area is $A = 2.7(0.9) = 2.43 \text{ m}^2$. The mass density is

$$\rho = \frac{m}{A} = \frac{80}{2.43} = 32.922 \text{ kg/m}^2.$$

The moment of inertia about the centroid of the rectangle is

$$I_{xc} = \rho \left(\frac{1}{12}\right) 2.7(0.9^3) = 5.4 \text{ kg-m}^2,$$

$$I_{yc} = \rho \left(\frac{1}{12}\right) 0.9(2.7^3) = 48.6 \text{ kg-m}^2.$$

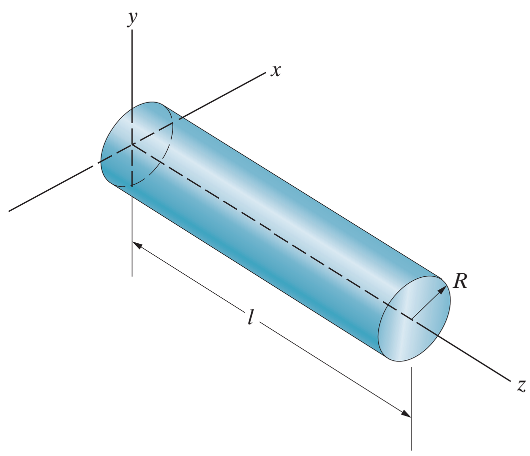
Use the parallel axis theorem:

$$I_{x \text{ axis}} = \rho A(0.6 + 0.45)^2 + I_{xc} = 93.6 \text{ kg-m}^2,$$

$$I_{y \text{ axis}} = \rho A(1.35 - 0.9)^2 + I_{yc} = 64.8 \text{ kg-m}^2.$$

$$I_{z \text{ axis}} = I_{x \text{ axis}} + I_{y \text{ axis}} = 158.4 \text{ kg-m}^2$$

Problem 8.122 The homogeneous cylinder has mass m, length l, and radius R. Use integration as described in Example 8.13 to determine its moment of inertia about the x axis.



Solution: The volume of the disk element is $\pi R^2 dz$ and its mass is $dm = \rho\pi R^2 dz$, where ρ is the density of the cylinder. From Appendix C, the moment of inertia of the disk element about the x' axis is

$$dI_{x'\text{axis}} = \frac{1}{4}dmR^2 = \frac{1}{4}(\rho\pi R^2 dz)R^2.$$

Applying the parallel-axis theorem, the moment of inertia of the disk element about the x axis is

$$dI_{x\text{axis}} = dI_{x'\text{axis}} + z^2 dm = \frac{1}{4}(\rho\pi R^2 dz)R^2 + z^2(\rho\pi R^2 dz)$$

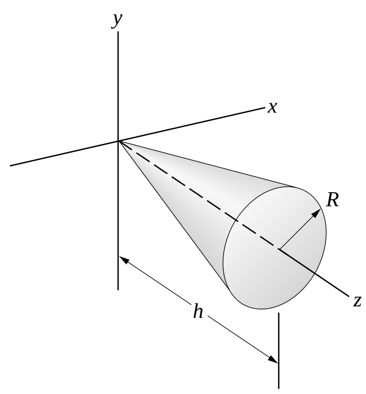
Integrating this expression from $z = 0$ to $z = l$ gives the moment of inertia of the cylinder about the x axis.

$$I_{x\text{axis}} = \int_0^l \left(\frac{1}{4}\rho\pi R^4 + \rho\pi R^2 z^2\right) dz = \frac{1}{4}\rho\pi R^4 l + \frac{1}{3}\rho\pi R^2 l^3.$$

In terms of the mass of the cylinder $m = \rho\pi R^2 l$,

$$\boxed{I_{x\text{axis}} = \frac{1}{4}mR^2 + \frac{1}{3}ml^2}$$

Problem 8.123 The homogenous cone is of mass m. Determine its moment of inertia about the z axis, and compare your result with the value given in Appendix C. (See Example 8.13.)



Solution: The differential mass

$$dm = \left(\frac{m}{V}\right)\pi r^2\, dz = \frac{3m}{R^2 h} r^2\, dz.$$

The moment of inertia of this disk about the z axis is $\frac{1}{2}mr^2$. The radius varies with z,

$$r = \left(\frac{R}{h}\right) z,$$

from which

$$I_{z\ \text{axis}} = \frac{3mR^2}{2h^5}\int_0^h z^4\, dz = \frac{3mR^2}{2h^5}\left[\frac{z^5}{5}\right]_0^h = \frac{3mR^2}{10}$$

Problem 8.124 Determine the moments of inertia of the homogenous cone in Problem 8.123 about the x and y axes, and compare your results with the values given in Appendix C.

Solution: The mass density is $\rho = \frac{m}{V} = \frac{3m}{\pi R^2 h}$. The differential element of mass is $dm = \rho\pi r^2\,dz$. The moment of inertia of this elemental disk about an axis through its center of mass, parallel to the x- and y-axes, is

$$dI_x = \left(\tfrac{1}{4}\right) r^2\,dm.$$

Use the parallel axis theorem,

$$I_x = \int_m \left(\tfrac{1}{4}\right) r^2\,dm + \int_m z^2\,dm.$$

Noting that $r = \frac{R}{h}z$, then

$$r^2\,dm = \rho\left(\frac{\pi R^4}{h^4}\right) z^4\,dz,$$

and $z^2\,dm = \rho\left(\frac{\pi R^2}{h^2}\right) z^4\,dz.$

Substitute:

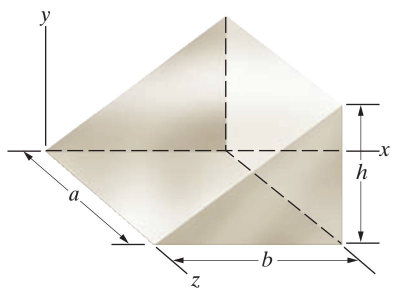
$$I_x = \rho\left(\frac{\pi R^4}{4h^4}\right)\int_0^h z^4\,dz + \rho\left(\frac{\pi R^2}{h^2}\right)\int_0^h z^4\,dz.$$

Integrating and collecting terms:

$$I_x = \left(\frac{3mR^2}{4h^5} + \frac{3m}{h^3}\right)\left[\frac{z^5}{5}\right]_0^h = m\left(\frac{3}{20}R^2 + \frac{3}{5}h^2\right).$$

By symmetry, $I_y = I_x$

Problem 8.125 The mass of the homogeneous wedge is m. Use integration as described in Example 8.13 to determine its moment of inertia about the z axis. (Your answer should be in terms of m, a, b, and h.)



Solution: Consider a triangular element of the wedge of thickness dz. The mass of the element is the product of the density ρ of the material and the volume of the element, $dm = \rho\frac{1}{2}bhdz$. The moments of inertia of the triangular element about the x' and y' axes are given by Eqs. (8.30) and (8.31) in terms of the mass of the element, its triangular area, and the moments of inertia of the triangular area:

$$dI_{x'\text{axis}} = \frac{dm}{A}I'_x = \frac{\rho\frac{1}{2}bhdz}{\frac{1}{2}bh}\left(\frac{1}{12}bh^3\right) = \frac{1}{12}rbh^3dz,$$

$$dI_{y'\text{axis}} = \frac{dm}{A}I'_y = \frac{\rho\frac{1}{2}bhdz}{\frac{1}{2}bh}\left(\frac{1}{4}hb^3\right) = \frac{1}{4}rhb^3dz,$$

The moment of inertia of this thin plate about the z axis is

$$dI_{z\text{axis}} = dI_{x'\text{axis}} + dI_{y'\text{axis}} = \frac{1}{12}\rho bh^3dz + \frac{1}{4}\rho hb^3dz.$$

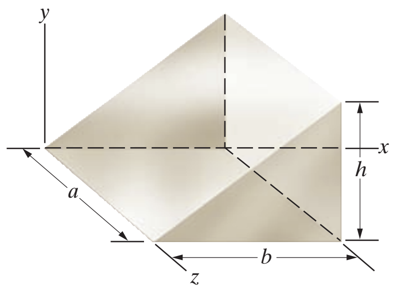
Integrating this expression from $z = 0$ to $z = a$ gives the moment of inertia of the wedge about the z axis:

$$I_{z\text{axis}} = \int_0^a \left(\frac{1}{12}\rho bh^3 + \frac{1}{4}\rho hb^3\right) dz = \frac{1}{12}\rho bh^3a + \frac{1}{4}\rho hb^3a.$$

In terms of the mass $m = \rho\frac{1}{2}bha$,

$$\boxed{I_{z\text{axis}} = \frac{1}{6}mh^2 + \frac{1}{2}mb^2.}$$

Problem 8.126 The mass of the homogeneous wedge is m. Use integration as described in Example 8.13 to determine its moment of inertia about the x axis. (Your answer should be in terms of m, a, b, and h.)



Solution: Consider a triangular element of the wedge of thickness dz. The mass of the element is the product of the density ρ of the material and the volume of the element, $dm = \rho\frac{1}{2}bhdz$. The moments of inertia of the triangular element about the x' axis is given by Eq. (8.30) in terms of the mass of the element, its triangular area, and the moments of inertia of the triangular area:

$$dI_{x'\text{axis}} = \frac{dm}{A}I_{x'} = \frac{\rho\frac{1}{2}bhdz}{\frac{1}{2}bh}\left(\frac{1}{36}bh^3\right) = \frac{1}{36}rbh^3dz,$$

Applying the parallel-axis theorem, the moment of inertia of the triangular element about the x axis is

$$dI_{x\text{axis}} = dI_{x'\text{axis}} + \left[z^2 + \left(\frac{1}{3}h\right)^2\right]dm$$

$$= \frac{1}{36}\rho bh^3dz + [z^2 + (\frac{1}{3}h)^2]\rho\frac{1}{2}bhdz = \frac{1}{12}\rho bh^3dz + \frac{1}{2}\rho bhz^2dz$$

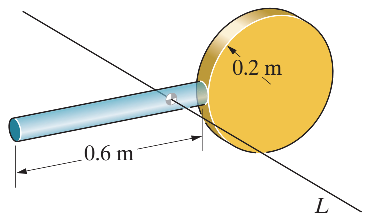
Integrating this expression from $z = 0$ to $z = a$ gives the moment of inertia of the wedge about the x axis:

$$I_{x\text{axis}} = \int_0^a \left(\frac{1}{12}\rho bh^3 + \frac{1}{2}\rho bhz^2\right)dz = \frac{1}{12}\rho bh^3a + \frac{1}{6}\rho bha^3.$$

In terms of the mass $m = \rho\frac{1}{2}bha$,

$$\boxed{I_{x\text{axis}} = \frac{1}{6}mh^2 + \frac{1}{3}ma^2.}$$

Problem 8.127 In Example 8.12, suppose that part of the 3-kg bar is sawed off so that the bar is 0.4 m long and its mass is 2 kg. Determine the moment of inertia of the composite object about the perpendicular axis L through the center of mass of the modified object.



Solution: The mass of the disk is 2 kg. Measuring from the left end of the rod, we locate the center of mass

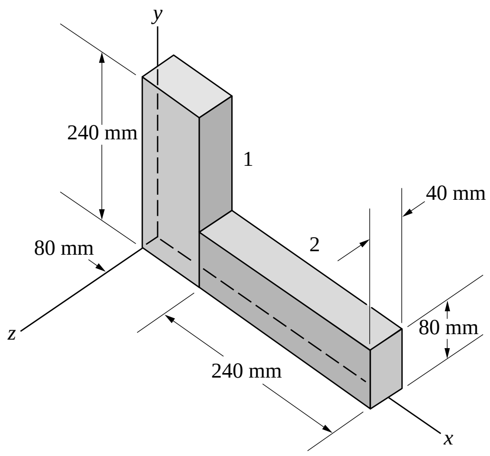
$$\bar{x} = \frac{(2\text{ kg})(0.2\text{ m}) + (2\text{ kg})(0.6\text{ m})}{(2\text{ kg}) + (2\text{ kg})} = 0.4\text{ m}.$$

The center of mass is located at the point where the rod and disk are connected. The moment of inertia is

$$I = \frac{1}{3}(2\text{ kg})(0.4\text{ m})^2 + \left\{\frac{1}{2}(2\text{ kg})(0.2\text{ m})^2 + (2\text{ kg})(0.2\text{ m})^2\right\}$$

$$\boxed{I = 0.227\text{ kg-m}^2.}$$

Problem 8.128 The L-shaped machine part is composed of two homogeneous bars. Bar 1 is tungsten alloy with mass density $14{,}000\ \text{kg/m}^3$, and bar 2 is steel with mass density $7800\ \text{kg/m}^3$. Determine its moment of inertia about the x axis.



Solution: The masses of the bars are

$$m_1 = (14{,}000)(0.24)(0.08)(0.04) = 10.75\ \text{kg}$$

$$m_2 = (7800)(0.24)(0.08)(0.04) = 5.99\ \text{kg}.$$

Using Appendix C and the parallel axis theorem the moments of inertia of the parts about the x axis are

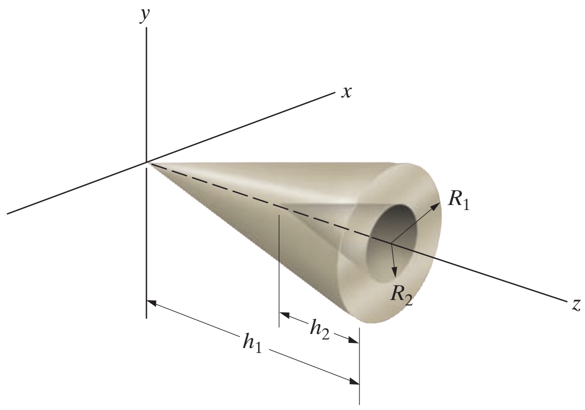
$$I_{(x\ \text{axis})1} = \frac{1}{12}m_1[(0.04)^2 + (0.24)^2] + m_1(0.12)^2 = 0.2079\ \text{kg-m}^2,$$

$$I_{(x\ \text{axis})2} = \frac{1}{12}m_2[(0.04)^2 + (0.08)^2] + m_2(0.04)^2 = 0.0136\ \text{kg-m}^2.$$

Therefore

$$I_{(x\ \text{axis})} = I_{(x\ \text{axis})1} + I_{(x\ \text{axis})2} = 0.221\ \text{kg-m}^2$$

Problem 8.129 The homogeneous object is a cone with a conical hole. The dimensions $R_1 = 2$ cm, $R_2 = 1$ cm, $h_1 = 6$ cm, and $h_2 = 3$ cm. It consists of an aluminum alloy with a density of 2700 kg/m^3. Determine its moment of inertia about the x axis.



Solution: The density of the material is

$$\rho = 2700 \text{ kg/m}^3.$$

The volume of the conical object *without the conical hole is*

$$V_1 = \frac{1}{3}\pi R_1^2 h_1 = \frac{1}{3}\pi(2 \text{ cm})^2(6 \text{ cm}) = 25.1 \text{ cm}^3.$$

The mass of the conical object without the conical hole is $m_1 = \rho V_1 = 0.0678$ kg. From Appendix C, the moment of inertia of the conical object without the conical hole about the x axis is

$$(I_x)_1 = m_1\left(\frac{3}{5}h_1^2 + \frac{3}{20}R_1^2\right)$$

$$= (0.0678 \text{ kg}) \left[\frac{3}{5}(6 \text{ cm})^2 + \frac{3}{20}(2 \text{ cm})^2\right] = 1.505 \text{ kg-cm}^2$$

The volume of the conical hole is

$$V_2 = \frac{1}{3}\pi R_2^2 h_2 = \frac{1}{3}\pi(1 \text{ cm})^2 (3 \text{ cm}) = 3.14 \text{ cm}^3$$

The mass of the material that would occupy the conical hole is $m_2 = \rho V_2 = 8.478 \times 10^{-3}$ kg. The z coordinate of the center of mass of the material that would occupy the conical hole is

$$\bar{z} = h_1 - h_2 + \frac{3}{4}h_2 = 6 \text{ cm} - 3 \text{ cm} + \frac{3}{4}(3 \text{ cm}) = 5.25 \text{ cm}.$$

Using Appendix C and applying the parallel-axis theorem, the moment of inertia about the x axis of the material that would occupy the conical hole is

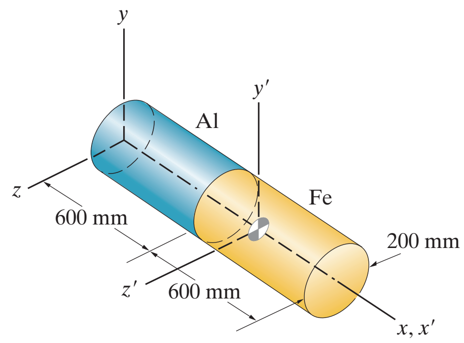
$$(I_x)_2 = m_2\left(\frac{3}{80}h_2^2 + \frac{3}{20}R_2^2\right) + \bar{z}^2 m_2 = 0.238 \text{ kg-cm}^2.$$

The moment of inertia of the conical object with the conical hole is

$$I_x = (I_x)_1 - (I_x)_2 = 1.267 \text{ kg-cm}^2.$$

$$\boxed{I_x = 1.267 \text{ kg-cm}^2.}$$

Problem 8.130 The circular cylinder is made of aluminum (Al) with density 2700 kg/m^3 and iron (Fe) with density 7860 kg/m^3. Determine its moments of inertia about the x' and y' axes.



Solution: We have $\rho_{\text{Al}} = 2700\ \text{kg/m}^3,\ \ \rho_{\text{Fe}} = 7860\ \text{kg/m}^3$

We first locate the center of mass

$$\bar{x} = \frac{\rho_{\text{Al}}(\pi[0.1\ \text{m}]^2[0.6\ \text{m}])(0.3\ \text{m}) + \rho_{\text{Fe}}(\pi[0.1\ \text{m}]^2[0.6\ \text{m}])(0.9\ \text{m})}{\rho_{\text{Al}}(\pi[0.1\ \text{m}]^2[0.6\ \text{m}]) + \rho_{\text{Fe}}(\pi[0.1\ \text{m}]^2[0.6\ \text{m}])}$$

$$= 0.747\ \text{m}$$

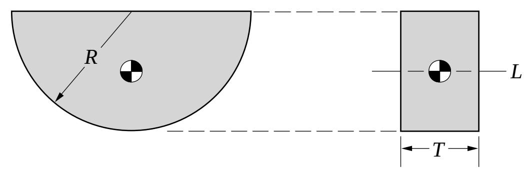
We also have the masses

$$m_{\text{Al}} = \rho_{\text{Al}}\pi(0.1\ \text{m})^2\ (0.6\ \text{m}),\ \ m_{\text{Fe}} = \rho_{\text{Fe}}\pi(0.1\ \text{m})^2\ (0.6\ \text{m})$$

Now find the moments of inertia

$$\boxed{\begin{aligned} I_{x'} &= \frac{1}{2}m_{\text{Al}}(0.1\ \text{m})^2 + \frac{1}{2}m_{\text{Fe}}(0.1\ \text{m})^2 = 0.995\ \text{kg}\,\text{m}^2 \\ I_{y'} &= m_{\text{Al}}\left(\frac{[0.6\ \text{m}]^2}{12} + \frac{[0.1\ \text{m}]^2}{4}\right) + m_{\text{Al}}(\bar{x} - 0.3\ \text{m})^2 \\ &\quad + m_{\text{Fe}}\left(\frac{[0.6\ \text{m}]^2}{12} + \frac{[0.1\ \text{m}]^2}{4}\right) + m_{\text{Fe}}(0.9\ \text{m} - \bar{x})^2 \\ &= 20.1\ \text{kg}\,\text{m}^2 \end{aligned}}$$

Problem 8.131 The homogenous half-cylinder is of mass m. Determine its moment of inertia about the axis L through its center of mass.



Solution: The centroid of the half cylinder is located a distance of $\left(\frac{4R}{3\pi}\right)$ from the edge diameter. The strategy is to use the parallel axis theorem to treat the moment of inertia of a complete cylinder as the sum of the moments of inertia for the two half cylinders. From Problem 8.118, the moment of inertia about the geometric axis for a cylinder is $I_{cL} = mR^2$, where m is one half the mass of the cylinder.

By the parallel axis theorem,

$$I_{cL} = 2\left(\left(\frac{4R}{3\pi}\right)^2 m + I_{hL}\right).$$

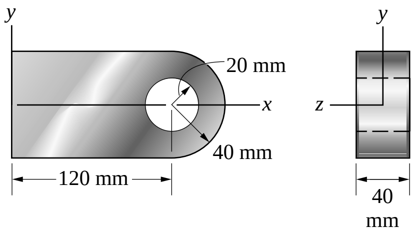
Solve

$$I_{hL} = \left(\frac{I_{cL}}{2} - \left(\frac{4R}{3\pi}\right)^2 m\right) = \left(\frac{mR^2}{2} - \left(\frac{16}{9\pi^2}\right)mR^2\right)$$

$$= mR^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$$

$$= mR^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right) = 0.31987\ mR^2 = 0.32\ mR^2$$

Problem 8.132 The homogeneous machine part is made of aluminum alloy with density $\rho = 2800$ kg/m^3. Determine its moment of inertia about the z axis.

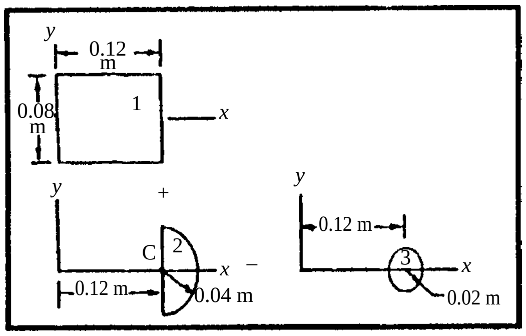


Solution: We divide the machine part into the 3 parts shown: (The dimension into the page is 0.04 m). The masses of the parts are

$$m_1 = (2800)(0.12)(0.08)(0.04) = 1.075 \text{ kg},$$

$$m_2 = (2800)\tfrac{1}{2}\pi(0.04)^2(0.04) = 0.281 \text{ kg},$$

$$m_3 = (2800)\pi(0.02)^2(0.04) = 0.141 \text{ kg}.$$



Using Appendix C and the parallel axis theorem the moment of inertia of part 1 about the z axis is

$$I_{(z\text{ axis})1} = \frac{1}{12}m_1[(0.08)^2 + (0.12)^2] + m_1(0.06)^2$$

$$= 0.00573 \text{ kg-m}^2.$$

The moment of inertia of part 2 about the axis through the center C that is parallel to the z axis is

$$\tfrac{1}{2}m_2R^2 = \tfrac{1}{2}m_2(0.04)^2.$$

The distance along the x axis from C to the center of mass of part 2 is

$$4(0.04)/(3\pi) = 0.0170 \text{ m}.$$

Therefore, the moment of inertia of part 2 about the z axis through its center of mass that is parallel to the axis is

$$\tfrac{1}{2}m_2(0.04)^2 - m_2(0.0170)^2 = 0.000144 \text{ kg-m}^2.$$

Using this result, the moment of inertia of part 2 about the z axis is

$$I_{(z\text{ axis})2} = 0.000144 + m_2(0.12 + 0.017)^2 = 0.00543 \text{ kg-m}^2.$$

The moment of inertia of the material that would occupy the hole 3 about the z axis is

$$I_{(z\text{ axis})3} = \tfrac{1}{2}m_3(0.02)^2 + m_3(0.12)^2 = 0.00205 \text{ kg-m}^2.$$

Therefore $I_{(z\text{ axis})} = I_{(z\text{ axis})1} + I_{(z\text{ axis})2} - I_{(z\text{ axis})3}$

$$= 0.00911 \text{ kg-m}^2.$$

Problem 8.133 Determine the moment of inertia of the machine part in Problem 8.132 about the x axis.

Solution: We divide the machine part into the 3 parts shown in the solution to Problem 8.132. Using Appendix C and the parallel axis theorem, the moments of inertia of the parts about the x axis are:

$$I_{(x\text{ axis})1} = \frac{1}{12}m_1[(0.08)^2 + (0.04)^2] = 0.0007168 \text{ kg-m}^2$$

$$I_{(x\text{ axis})2} = m_2\left[\frac{1}{12}(0.04)^2 + \frac{1}{4}(0.04)^2\right]$$

$$= 0.0001501 \text{ kg-m}^2$$

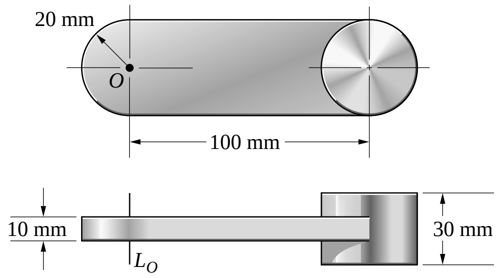
$$I_{(x\text{ axis})3} = m_3\left[\frac{1}{12}(0.04)^2 + \frac{1}{4}(0.02)^2\right]$$

$$= 0.0000328 \text{ kg-m}^2.$$

Therefore, $I_{(x\text{ axis})} = I_{(x\text{ axis})1} + I_{(x\text{ axis})_2} - I_{(x\text{ axis})3}$

$$= 0.000834 \text{ kg-m}^2.$$

Problem 8.134 The object consists of steel of density $\rho = 7800$ kg/m^3. Determine its moment of inertia about the axis L_O.



Solution: Divide the object into four parts: Part (1) The semi-cylinder of radius $R = 0.02$ m, height $h_1 = 0.01$ m.

Part (2): The rectangular solid $L = 0.1$ m by $h_2 = 0.01$ m by $w = 0.04$ m. Part (3): The semi-cylinder of radius $R = 0.02$ m, $h_1 = 0.01$ m. Part (4) The cylinder of radius $R = 0.02$ m, height $h = 0.03$ m.

Part (1)

$$m_1 = \frac{\rho \pi R^2 h_1}{2} = 0.049 \text{ kg},$$

$$I_1 = \frac{m_1 R^2}{4} = 4.9 \times 10^{-6} \text{ kg-m}^2,$$

Part (2):

$$m_2 = \rho w L h_2 = 0.312 \text{ kg},$$

$$I_2 = \left(\frac{1}{12}\right) m_2 (L^2 + w^2) + m_2 \left(\frac{L}{2}\right)^2 = 0.00108 \text{ kg-m}^2.$$

Part (3)

$$m_3 = m_1 = 0.049 \text{ kg},$$

$$I_3 = -\left(\frac{4R}{3\pi}\right)^2 m_2 + I_1 + m_3 \left(L - \frac{4R}{3\pi}\right)^2 = 0.00041179 \text{ kg m}^2.$$

Part (4)

$$m_4 = \rho \pi R^2 h = 0.294 \text{ kg},$$

$$I_4 = \left(\tfrac{1}{2}\right) m_4 (R^2) + m_4 L^2 = 0.003 \text{ kg m}^2.$$

The composite:

$$I_{Lo} = I_1 + I_2 - I_3 + I_4 = 0.003674 \text{ kg m}^2$$

Problem 8.135 Determine the moment of inertia of the object in Problem 8.134 about the axis through the center of mass of the object parallel to L_O.

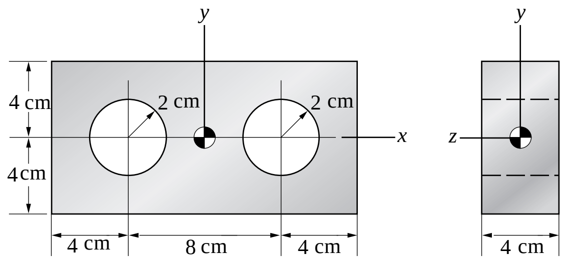
Solution: The center of mass is located relative to L_O

$$\mathbf{x} = \frac{m_1\left(-\dfrac{4R}{3\pi}\right) + m_2(0.05) - m_3\left(0.1 - \dfrac{4R}{3\pi}\right) + m_4(0.1)}{m_1 + m_2 - m_3 + m_4}$$

$$= 0.066 \text{ m},$$

$$I_c = -\mathbf{x}^2 m + I_{Lo} = -0.00265 + 0.00367 = 0.00102 \text{ kg m}^2$$

Problem 8.136 The thick plate consists of steel of density $\rho = 7800$ kg/m^3. Determine its moment of inertia about the z axis.



Solution: Divide the object into three parts: Part (1) the rectangle 8 cm by 16 cm, Parts (2) & (3) the cylindrical cut outs. Part (1):

$m_1 = \rho 0.08(0.16)(0.04) = 3.994$ kg.

$$I_1 = \left(\frac{1}{12}\right) m_1(0.16^2 + 0.08^2) = 0.01065 \text{ kg-m}^2.$$

Part (2):

$m_2 = \rho\pi(0.02^2)(0.04) = 0.3921$ kg,

$$I_2 = \frac{m_2(0.02^2)}{2} + m_2(0.04^2) = 0.0007057 \text{ kg-m}^2.$$

Part (3):

$m_3 = m_2 = 0.3921$ kg,

$I_3 = I_2 = 0.0007057$ kg-m^2.

The composite:

$I_{z\ \text{axis}} = I_1 - 2I_2 = 0.00924$ kg-m^2

Problem 8.137 Determine the moment of inertia of the plate in Problem 8.136 about the x axis.

Solution: Use the same divisions of the object as in Problem 8.136.

Part (1):

$$I_{1x\ \text{axis}} = \left(\frac{1}{12}\right) m_1(0.08^2 + 0.04^2) = 0.002662 \text{ kg-m}^2,$$

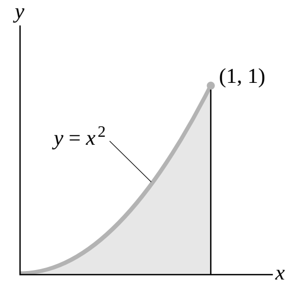
Part (2):

$$I_{2x\ \text{axis}} = \left(\frac{1}{12}\right) m_2(3(0.02^2) + 0.04^2) = 9.148 \times 10^{-5} \text{ kg-m}^2.$$

The composite:

$I_{x\ \text{axis}} = I_{1x\ \text{axis}} - 2I_{2x\ \text{axis}} = 0.00248$ kg-m^2

Problem 8.138 Determine I_y and k_y.



Solution:

$$dA = dx\,dy A = \int_0^1 dx \int_0^{x^2} dy = \int_0^1 x^2\,dx = \frac{1}{3}.$$

$$I_y = \int_A x^2\,dA = \int_0^1 x^2\,dx \int_0^{x^2} dy = \int_0^1 x^4\,dx = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3}{5}}$$

Problem 8.139 Determine I_x and k_x.

Solution: (See figure in Problem 8.138.) $dA = dx\,dy$,

$$I_x = \int_A y^2\,dA = \int_0^1 dx \int_0^{x^2} y^2\,dy = \frac{1}{3}\int_0^1 x^6\,dx$$

$$= \left(\frac{1}{21}\right)[x^7]_0^1 = \frac{1}{21}$$

$$k_x = \sqrt{\frac{I_x}{A}} = \frac{1}{\sqrt{7}}$$

Problem 8.140 Determine J_O and k_O.

Solution: (See figure in Problem 8.138.)

$$J_O = I_x + I_y = \frac{1}{5} + \frac{1}{21} = \frac{26}{105},$$

$$k_O = \sqrt{k_x^2 + k_y^2} = \sqrt{\frac{3}{5} + \frac{1}{7}} = \sqrt{\frac{26}{35}}$$

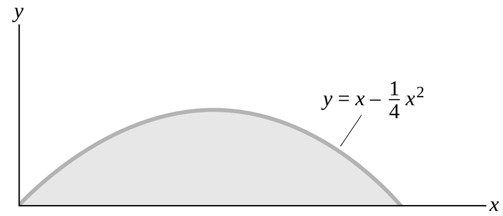
Problem 8.141 Determine I_{xy}.

Solution: (See figure in Problem 8.138.) $dA = dx\,dy$

$$I_{xy} = \int_A xy\,dA = \int_0^1 x\,dx \int_0^{x^2} y\,dy$$

$$= \frac{1}{2}\int_0^1 x^5\,dx = \frac{1}{12}[x^6]_0^1 = \frac{1}{12}$$

Problem 8.142 Determine I_y and k_y.



Solution: By definition,

$$I_y = \int_A x^2\, dA.$$

The element of area is $dA = dx\,dy$. The limits on the variable x are $0 \le x \le 4$. The area is

$$A = \int_0^4 dx \int_0^{x-x^2/4} dy = \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = 2.6667$$

$$I_y = \int_0^4 x^2\,dx \int_0^{x-x^2/4} dy = \int_0^4 \left(x - \frac{x^2}{4}\right) x^2\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{20}\right]_0^4 = 12.8$$

from which

$$k_y = \sqrt{\frac{I_y}{A}} = 2.19$$

Problem 8.143 Determine I_x and k_x.

Solution: By definition,

$$I_x = \int_A y^2\, dA,$$

from which

$$I_x = \int_0^4 dx \int_0^{x-x^2/4} y^2\,dy = \left(\frac{1}{3}\right)\int_0^4 \left(x - \frac{x^2}{4}\right)^3 dx$$

$$I_x = \left(\frac{1}{3}\right)\left[\frac{x^4}{4} - \frac{3}{20}x^5 + \frac{3}{96}x^6 - \frac{x^7}{448}\right]_0^4 = 0.6095.$$

From Problem 8.142,

$$A = 2.667,\ \ k_x = \sqrt{\frac{I_x}{A}} = 0.4781$$

Problem 8.144 Determine I_{xy}.

Solution:

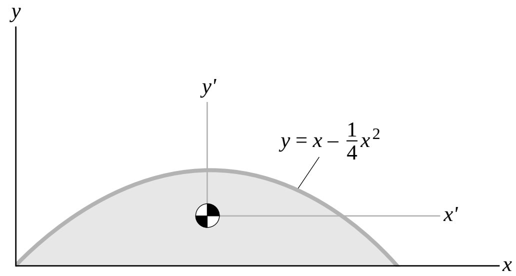
$$I_{xy} = \int_A xy\, dA,$$

$$= \int_0^4 x\,dx \int_0^{x-x^2/4} y\,dy$$

$$= \left(\frac{1}{2}\right)\int_0^4 \left(x - \frac{x^2}{4}\right)^2 x\,dx$$

$$= \left(\frac{1}{2}\right)\left[\frac{x^4}{4} - \frac{x^5}{10} + \frac{x^6}{96}\right]_0^4 = 2.1333$$

Problem 8.145 Determine $I_{y'}$ and $k_{y'}$.



Solution: The limits on the variable x are $0 \le x \le 4$. By definition,

$$A\mathbf{y} = \int_A y\,dA = \int_0^4 dx \int_0^{x-x^2/4} y\,dy$$

$$= \left(\frac{1}{2}\right)\int_0^4 \left(x - \frac{x^2}{4}\right)^2 dx$$

$$= \left(\frac{1}{2}\right)\left[\frac{x^3}{3} - \frac{x^4}{8} + \frac{x^5}{80}\right]_0^4 = 1.06667.$$

From Problem 8.142 the area is $A = 2.667$, from which $\mathbf{y} = 0.3999 = 0.4$. Similarly,

$$A\mathbf{x} = \int_0^4 x\,dx \int_0^{X-X^2/4} dy$$

$$= \int_0^4 x\left(x - \frac{x^2}{4}\right) dx = \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_0^4 = 5.3333,$$

from which $\mathbf{x} = 1.9999 = 2$. The area moment of inertia is $I_{yy} = -\mathbf{x}^2 A + I_y$. Using the result of Problem 8.142, $I_y = 12.8$, from which the area moment of inertia about the centroid is

$$I_{y'} = -10.6666 + 12.8 = 2.133$$

and $k_{y'} = \sqrt{\dfrac{I_{y'}}{A}} = 0.8944$

Problem 8.146 Determine $I_{x'}$ and $k_{x'}$.

Solution: Using the results of Problems 8.143 and 8.145, $I_x = 0.6095$ and $\mathbf{y} = 0.4$. The area moment of inertia about the centroid is

$$I_{x'} = -\mathbf{y}^2 A + I_x = 0.1828$$

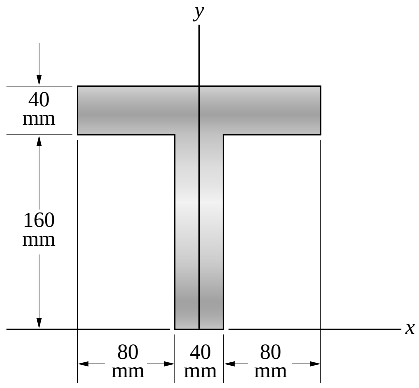
and $k_{x'} = \sqrt{\dfrac{I_{x'}}{A}} = 0.2618$

Problem 8.147 Determine $I_{x'y'}$.

Solution: From Problems 8.143 and 8.144, $I_{xy} = 2.133$ and $\mathbf{x} = 2$, $\mathbf{y} = 0.4$. The product of the moment of inertia about the centroid is

$$I_{x'y'} = -\mathbf{xy}A + I_{xy} = -2.133 + 2.133 = 0$$

Problem 8.148 Determine I_y and k_y.



Solution: Divide the section into two parts: Part (1) is the upper rectangle 40 mm by 200 mm, Part (2) is the lower rectangle, 160 mm by 40 mm.

Part (1) $A_1 = 0.040(0.200) = 0.008\ \text{m}^2$,

$\mathbf{y}_1 = 0.180$ m

$\mathbf{x}_1 = 0$,

$$I_{y1} = \left(\frac{1}{12}\right) 0.04(0.2)^3 = 2.6667 \times 10^{-5}\text{m}^4.$$

Part (2): $A_2 = (0.04)(0.16) = 0.0064\ \text{m}^2$,

$\mathbf{y}_2 = 0.08$ m,

$\mathbf{x}_2 = 0$,

$$I_{y2} = \left(\frac{1}{12}\right) (0.16)(0.04)^3 = 8.5 \times 10^{-7}\ \text{m}^4.$$

The composite:

$$A = A_1 + A_2 = 0.0144\ \text{m}^2,$$

$$I_y = I_{y1} + I_{y2},$$

$$I_y = 2.752 \times 10^{-5}\text{m}^4 = 2.752 \times 10^7\ \text{mm}^4,$$

and $k_y = \sqrt{\dfrac{I_y}{A}} = 0.0437$ m $= 43.7$ mm

Problem 8.149 Determine I_x and k_x for the area in Problem 8.148.

Solution: Use the results in the solution to Problem 8.148. Part (1)

$A_1 = 0.040(0.200) = 0.008\ \text{m}^2$,

$\mathbf{y}_1 = 0.180$ m,

$$I_{x1} = \left(\frac{1}{12}\right) 0.2(0.04^3) + (0.18)^2 A_1 = 2.603 \times 10^{-4}\text{m}^4.$$

Part (2):

$A_2 = (0.04)(0.16) = 0.0064\ \text{m}^2$,

$\mathbf{y}_2 = 0.08$ m,

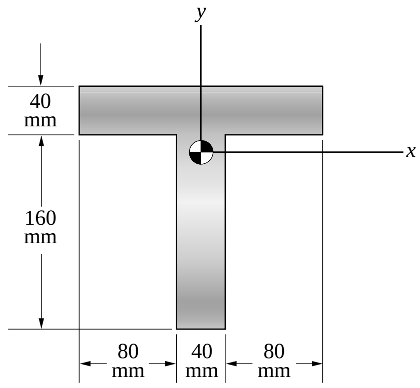
$$I_{x2} = \left(\frac{1}{12}\right) (0.04)(0.16)^3 + (0.08)^2 A_2 = 5.461 \times 10^{-5}\ \text{m}^4.$$

The composite: $A = A_1 + A_2 = 0.0144\ \text{m}^2$, The area moment of inertia about the x axis is

$$I_x = I_{x1} + I_{x2} = 3.15 \times 10^{-4}\ \text{m}^4 = 3.15 \times 10^8\ \text{mm}^4,$$

and $k_x = \sqrt{\dfrac{I_x}{A}} = 0.1479$ m $= 147.9$ mm

Problem 8.150 Determine I_x and k_x.



Solution: Use the results of the solutions to Problems 8.148–8.149. The centroid is located relative to the base at

$$\mathbf{x}_c = \frac{\mathbf{x}_1 A_1 + \mathbf{x}_2 A_2}{A} = 0,$$

$$\mathbf{y}_c = \frac{\mathbf{y}_1 A_1 + \mathbf{y}_2 A_2}{A} = 0.1356 \text{ m}.$$

The moment of inertia about the x axis is

$$I_{xc} = -\mathbf{y}_C^2 A + I_X = 5.028 \times 10^7 \text{ mm}^4$$

and $k_{xc} = \sqrt{\dfrac{I_{xc}}{A}} = 59.1$ mm

Problem 8.151 Determine J_O and k_O for the area in Problem 8.150.

Solution: Use the results of the solutions to Problems 8.148–8.149. The area moments of inertia about the centroid are

$$I_{xc} = 5.028 \times 10^{-5} \text{ m}^4$$

and $I_{yc} = I_y = 2.752 \times 10^{-5}$ m^4,

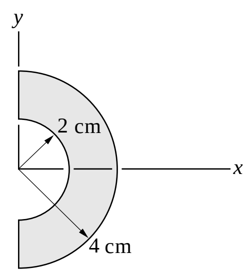
from which

$$J_O = I_{xc} + I_{yc} = 7.78 \times 10^{-5} \text{ m}^4 = 7.78 \times 10^7 \text{ mm}^4$$

and $k_O = \sqrt{\dfrac{J_O}{A}} = 0.0735$ m

$= 73.5$ mm

Problem 8.152 Determine I_y and k_y.



Solution: For a semicircle about a diameter:

$$I_{yy} = I_{xx} = \left(\frac{1}{8}\right)\pi R^4,$$

$$I_y = \left(\frac{1}{8}\right)\pi(4)^4 - \left(\frac{1}{8}\right)\pi(2)^4 = \frac{\pi}{8}(4^4 - 2^4) = 94.25 \text{ cm}^4,$$

$$k_y = \sqrt{\frac{2I_y}{\pi(4^2 - 2^2)}} = 2.236 \text{ cm}$$

Problem 8.153 Determine J_O and k_O. for the area in Problem 8.152.

Solution: For a semicircle:

$$I_{yy} = I_{xx} = \left(\frac{1}{8}\right)\pi R^4.$$

$$I_x = \frac{\pi}{8}(4^4 - 2^4) = 94.248 \text{ cm}^4.$$

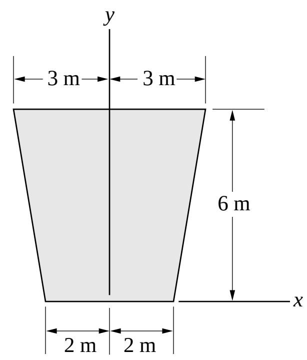
$$k_x = \sqrt{\frac{2I_x}{\pi(4^2 - 2^2)}} = 2.236 \text{ cm}.$$

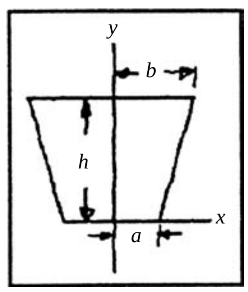
Also use the solution to Problem 8.152.

$$J_O = I_x + I_y = 2(94.248) = 188.5 \text{ cm}^4$$

$$k_O = \sqrt{\frac{2J_O}{\pi(4^2 - 2^2)}} = 3.16 \text{ cm}$$

Problem 8.154 Determine I_x and k_x.





Solution: Break the area into three parts: Part (1) The rectangle with base $2a$ and altitude h; Part (2) The triangle on the right with base $(b - a)$ and altitude h, and Part (3) The triangle on the left with base $(b - a)$ and altitude h. Part (1) The area is

$$A_1 = 2ah = 24 \text{ m}^2.$$

The centroid is

$$\mathbf{x}_1 = 0$$

and $\mathbf{y}_1 = \dfrac{h}{2} = 3$ m.

The area moment of inertia about the centroid is

$$I_{xc1} = \left(\frac{1}{12}\right)(2a)h^3 = \left(\frac{1}{6}\right)ah^3 = 72 \text{ m}^4.$$

Part (2): $A_2 = \left(\dfrac{1}{2}\right)h(b - a) = 3 \text{ m}^2$,

$$\mathbf{x}_2 = a + \frac{b - a}{3} = 2.3333 \text{ m},$$

$$\mathbf{y}_2 = \left(\frac{2}{3}\right)h = 4 \text{ m},$$

$$I_{xc2} = \left(\frac{1}{36}\right)(b - a)h^3 = 6 \text{ m}^4.$$

Part (3): $A_3 = A_2$,

$$\mathbf{x}_3 = -\mathbf{x}_2,\ \mathbf{y}_3 = \mathbf{y}_2,\ I_{xc3} = I_{xc2}.$$

The composite area is

$$A = A_1 + A_2 + A_2 = 30 \text{ m}^2.$$

The composite moment of inertia

$$I_x = (\mathbf{y}_1)^2 A_1 + I_{xc1} + (\mathbf{y}_2)^2 A_2 + I_{xc2} + (\mathbf{y}_3)^2 A_3 + I_{xc3},$$

$$I_x = 396 \text{ m}^4$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{396}{30}} = 3.633 \text{ m}$$

Problem 8.155 Determine I_y and k_y for the area in Problem 8.154.

Solution: Divide the area as in the solution to Problem 8.154. Part (1) The area is $A_1 = 2ah = 24\ \text{m}^2$. The centroid is $\mathbf{x}_1 = 0$ and $\mathbf{y}_1 = \frac{h}{2} = 3$ m. The area moment of inertia about the centroid is

$$I_{yc1} = \left(\frac{1}{12}\right) h(2a)^3 = \left(\frac{2}{3}\right) ha^3 = 32\ \text{m}^4$$

Part (2): $A_2 = \left(\frac{1}{2}\right) h(b - a) = 3\ \text{m}^2$,

$$\mathbf{x}_2 = a + \frac{b - a}{3} = 2.3333\ \text{m},$$

$$\mathbf{y}_2 = \left(\frac{2}{3}\right) h = 4\ \text{m},$$

$$I_{yc2} = \left(\frac{1}{36}\right) h(b - a)^3 = 0.1667\ \text{m}^4.$$

Part (3): $A_3 = A_2$,

$$\mathbf{x}_3 = -\mathbf{x}_2,\ \mathbf{y}_3 = \mathbf{y}_2,\ I_{yc3} = I_{yc2}.$$

The composite area is

$$A = A_1 + A_2 + A_2 = 30\ \text{m}^2.$$

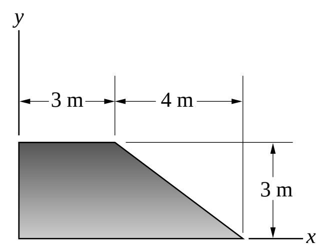
The composite moment of inertia,

$$I_y = \mathbf{x}_1^2 A_1 + I_{yc1} + \mathbf{x}_2^2 A_2 + I_{yc2} + \mathbf{x}_3^2 A_3 + I_{yc3},$$

$$I_y = 65\ \text{m}^4$$

and $k_y = \sqrt{\dfrac{I_y}{A}} = 1.472$ m

Problem 8.156 The moments of inertia of the area are $I_x = 36\ \text{m}^4$, $I_y = 145\ \text{m}^4$, and $I_{xy} = 44.25\ \text{m}^4$. Determine a set of principal axes and the principal moment of inertia.



Solution: The principal angle is

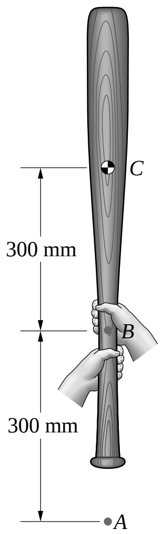
$$\theta = \left(\frac{1}{2}\right) \tan^{-1}\left(\frac{2I_{xy}}{I_y - I_x}\right) = 19.54^\circ.$$

The principal moments of inertia are

$$I_{xP} = I_x \cos^2\theta - 2I_{xy}\sin\theta\cos\theta + I_y \sin^2\theta = 20.298 = 20.3\ \text{m}^4$$

$$I_{yP} = I_x \sin^2\theta + 2I_{xy}\sin\theta\cos\theta + I_y \cos^2\theta = 160.70\ \text{m}^4$$

Problem 8.157 The moment of inertia of the 0.88 kg bat about a perpendicular axis through point B is 0.122 kg-m^2. What is the bat's moment of inertia about a perpendicular axis through point A? (Point A is the bat's "instantaneous center," or center of rotation, at the instant shown.)



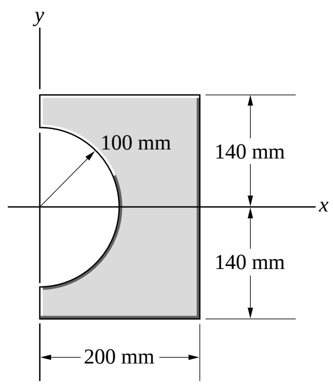
Solution:

Use the parallel axis theorem to obtain the moment of inertia about the center of mass C, and then use the parallel axis theorem to translate to the point A.

$$I_C = -(0.3)^2\ m = 0.04280\ \text{kg-m}^2$$

$$I_A = (0.3 + 0.35)^2\ m + 0.04280 = 0.415\ \text{kg-m}^2$$

Problem 8.158 The mass of the thin homogenous plate is 4 kg. Determine its moment of inertia about the y axis.



Solution: Divide the object into two parts: Part (1) is the semi-circle of radius 100 mm, and Part (2) is the rectangle 200 mm by 280 mm. The area of Part (1)

$$A_1 = \frac{\pi R^2}{2} = 15708\ \text{mm}^2.$$

The area of Part (2) is

$$A_2 = 280(200) = 56000\ \text{mm}^2.$$

The composite area is $A = A_2 - A_1 = 40292\ \text{mm}^2$. The area mass density is

$$\rho = \frac{4}{A} = 9.9275 \times 10^{-5}\ \text{kg/mm}^2.$$

For Part (1) $\mathbf{x}_1 = \mathbf{y}_1 = 0$,

$$I_{y1} = \rho\left(\frac{1}{8}\right)\pi R^4 = 3898.5\ \text{kg-mm}^2.$$

For Part (2) $\mathbf{x}_2 = 100$ mm.

$$I_{y2} = \mathbf{x}_2^2 \rho A_2 + \rho\left(\frac{1}{12}\right)(280)(200^3) = 74125.5\ \text{kg-mm}^2.$$

The composite:

$$I_y = I_{y2} - I_{y1} = 70226\ \text{kg-mm}^2 = 0.070226\ \text{kg-m}^2$$

Problem 8.159 Determine the moment of inertia of the plate in Problem 8.158 about the z axis.

Solution: Use the same division of the parts and the results of the solution to Problem 8.158. For Part (1),

$$I_{x1} = \rho \left(\frac{1}{8}\right) \pi R^4 = 3898.5 \text{ kg-mm}^2.$$

For Part (2)

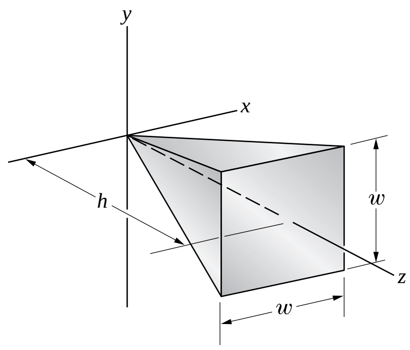
$$I_{x2} = \rho \left(\frac{1}{12}\right) (200)(280^3) = 36321.5 \text{ kg-mm}^2.$$

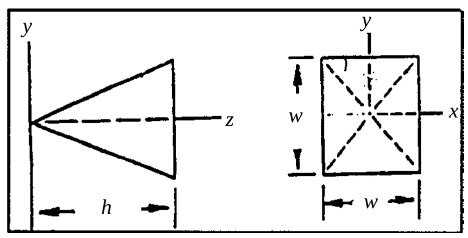
The composite: $I_x = I_{x2} - I_{x1} = 32423$ kg-mm^2,
from which, using the result of the solution to Problem 8.158

$$I_z = I_x + I_y = 32422 + 70226 = 102649 \text{ kg-mm}^2$$

$$= 0.10265 \text{ kg-m}^2$$

Problem 8.160 The homogenous pyramid is of mass m. Determine its moment of inertia about the z axis.





Solution: The mass density is

$$\rho = \frac{m}{V} = \frac{3m}{w^2 h}.$$

The differential mass is $dm = \rho\omega^2\, dz$. The moment of inertia of this element about the z axis is

$$dI_Z = \left(\frac{1}{6}\right) \omega^2\, dm.$$

Noting that $\omega = \dfrac{wz}{h}$, then

$$dI_z = \rho \left(\frac{w^4}{6h^4}\right) z^4\, dz = \frac{mw^2}{2h^5} z^4\, dz.$$

Integrating:

$$I_{z \text{ axis}} = \left(\frac{mw^2}{2h^5}\right) \int_0^h z^4\, dz = \frac{1}{10} mw^2$$

Problem 8.161 Determine the moment of inertia of the homogenous pyramid in Problem 8.160 about the x and y axes.

Solution: Use the results of the solution of Problem 8.160 for the mass density. The elemental disk is $dm = \rho\omega^2\, dz$. The moment of inertia about an axis through its center of mass parallel to the x axis is

$$dI_X = \left(\frac{1}{12}\right) \omega^2\, dm.$$

Use the parallel axis theorem:

$$I_{x \text{ axis}} = \left(\frac{1}{12}\right) \int_m \omega^2\, dm + \int_m z^2\, dm.$$

Noting that $\omega = \dfrac{w}{h} z$, the integral is

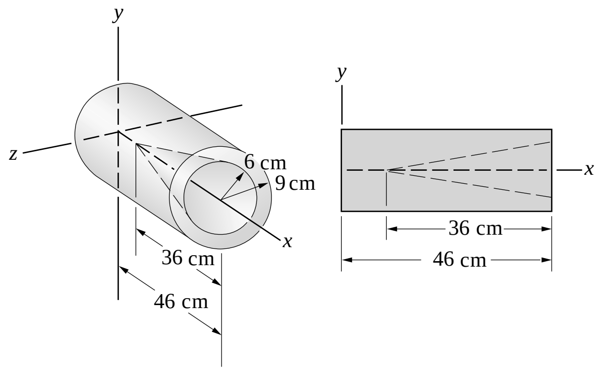
$$I_{x \text{ axis}} = \frac{\rho w^4}{12h^4} \int_0^h z^4\, dz + \frac{\rho w^2}{h^2} \int_0^h z^4\, dz.$$

Integrating and collecting terms

$$I_{x \text{ axis}} = m \left(\frac{1}{20} w^2 + \frac{3}{5} h^2\right).$$

By symmetry, $I_{y \text{ axis}} = I_{x \text{ axis}}$

Problem 8.162 The homogenous object weighs 400 N. Determine its moment of inertia about the x axis.



Solution: The volumes are

$$V_{cyl} = (46)\pi(9)^2 = 11{,}706 \text{ cm}^3,$$

$$V_{\text{cone}} = \tfrac{1}{3}\pi(6)^2(36) = 1357 \text{ cm}^3,$$

$$\text{so } V = V_{cyl} - V_{\text{cone}} = 10{,}348 \text{ cm}^3.$$

The masses of the *solid* cylinder and the material that would occupy the conical hole are

$$m_{cyl} = \left(\frac{V_{cyl}}{V}\right)\left(\frac{400}{9.81}\right) = 46.122 \text{ kg},$$

$$m_{\text{cone}} = \left(\frac{V_{\text{cone}}}{V}\right)\left(\frac{400}{9.81}\right) = 5.348 \text{ kg}.$$

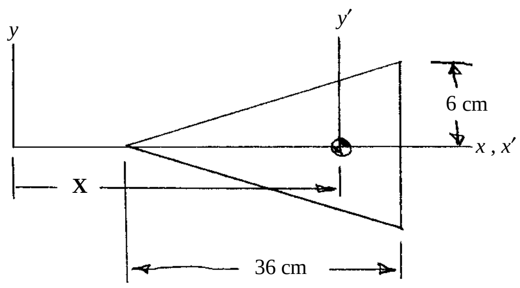
Using results from Appendix C,

$$I_{(x \text{ axis})} = \frac{1}{2}\,\mathrm{m}_{cyl}(9)^2 - \frac{3}{10}\,\mathrm{m}_{\text{cone}}(6)^2$$

$$= 1810 \text{ kg-cm}^2$$

Problem 8.163 Determine the moments of inertia of the object in Problem 8.162 about the y and z axes.

Solution: See the solution of Problem 8.162. The position of the center of mass of the material that would occupy the conical hole is



$$\mathbf{x} = (46 - 36) + \frac{3}{4}(36) = 37 \text{ cm}.$$

From Appendix C,

$$I_{(y' \text{ axis})\text{cone}} = m_{\text{cone}}\left[\frac{3}{80}(36)^2 + \frac{3}{20}(6)^2\right]$$

$$= 288.77 \text{ kg-cm}^2.$$

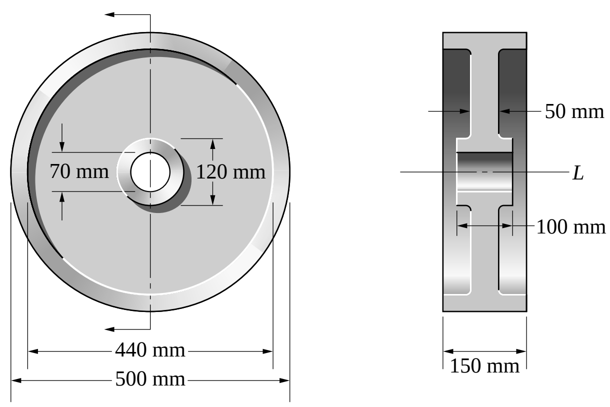
The moment of inertia about the y axis for the composite object is

$$I_{(y \text{ axis})} = m_{cyl}\left[\tfrac{1}{3}(46)^2 + \tfrac{1}{4}(9)^2\right]$$

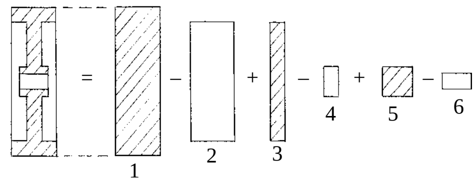
$$- \left(I_{(y' \text{ axis})\text{cone}} + \mathbf{x}^2 m_{\text{cone}}\right)$$

$$= 25856 \text{ kg-cm}^2$$

Problem 8.164 Determine the moment of inertia of the 14-kg flywheel about the axis L.



Solution: The flywheel can be treated as a composite of the objects shown:



The volumes are

$$V_1 = (150)\pi(250)^2 = 294.5 \times 10^5 \text{ mm}^3,$$

$$V_2 = (150)\pi(220)^2 = 228.08 \times 10^5 \text{ mm}^3,$$

$$V_3 = (50)\pi(220)^2 = 76.03 \times 10^5 \text{ mm}^3,$$

$$V_4 = (50)\pi(60)^2 = 5.65 \times 10^5 \text{ mm}^3,$$

$$V_5 = (100)\pi(60)^2 = 11.31 \times 10^5 \text{ mm}^3,$$

$$V_6 = (100)\pi(35)^2 = 3.85 \times 10^5 \text{ mm}^3.$$

The volume

$$V = V_1 - V_2 + V_3 - V_4 + V_5 - V_6$$

$$= 144.3 \times 10^5 \text{ mm}^3,$$

so the density is

$$\delta = \frac{14}{V} = 9.704 \times 10^{-7} \text{ kg/mm}^3.$$

The moment of inertia is

$$I_L = \tfrac{1}{2}\delta V_1(250)^2 - \tfrac{1}{2}\delta V_2(220)^2$$

$$+ \tfrac{1}{2}\delta V_3(220)^2 - \tfrac{1}{2}\delta V_4(60)^2$$

$$+ \tfrac{1}{2}\delta V_5(60)^2 - \tfrac{1}{2}\delta V_6(35)^2$$

$$= 536{,}800 \text{ kg-mm}^2$$

$$= 0.5368 \text{ kg-m}^2.$$