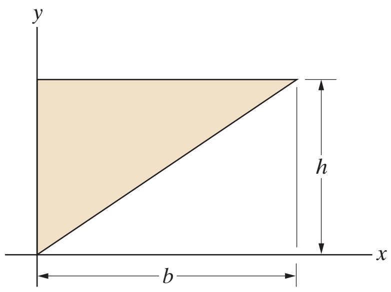
Problem 7.1 In Active Example 7.1, Suppose that the triangular area is oriented as shown. Use integration to determine the x and y coordinates of its centroid. (Notice that you already know the answers based on the results of Active Example 7.1.)



Solution: The height of the vertical strip is $h - (h/b)\,x$ so the area is $dA = \left(h - \dfrac{h}{b}x\right) dx$. Use this expression to evaluate Eq. (7.6).

The x coordinate of the centroid is

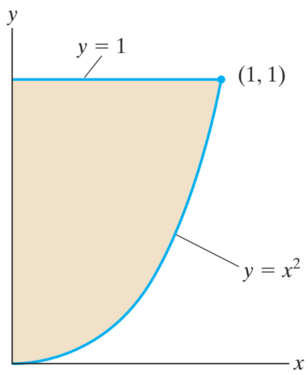
$$\bar{x} = \frac{\displaystyle\int_A x\,dA}{\displaystyle\int_A dA} = \frac{\displaystyle\int_0^b x\left(h - \frac{h}{b}x\right) dx}{\displaystyle\int_0^b \left(h - \frac{h}{b}x\right) dx} = \frac{h\left[\dfrac{x^2}{2} - \dfrac{x^3}{3b}\right]_0^b}{h\left[x - \dfrac{x^2}{2b}\right]_0^b} = \frac{b}{3}$$

The y coordinate of the midpoint of the vertical strip is $h - \dfrac{1}{2}\left(h - \dfrac{h}{b}x\right) = \dfrac{1}{2}\left(h + \dfrac{h}{b}x\right)$. We let this value be the value of y in Eq. (7.7):

$$\bar{y} = \frac{\displaystyle\int_A y\,dA}{\displaystyle\int_A dA} = \frac{\displaystyle\int_0^b \frac{1}{2}\left(h + \frac{h}{b}x\right)\left(h - \frac{h}{b}x\right) dx}{\displaystyle\int_0^b \left(h - \frac{h}{b}x\right) dx} = \frac{\dfrac{h^2}{2}\left[x - \dfrac{x^3}{3b^2}\right]_0^b}{h\left[x - \dfrac{x^2}{2b}\right]_0^b} = \frac{2h}{3}$$

$$\boxed{\bar{x} = \frac{b}{3}, \quad \bar{y} = \frac{2h}{3}}$$

Problem 7.2 In Example 7.2, suppose that the area is redefined as shown. Determine the x coordinate of the centroid.



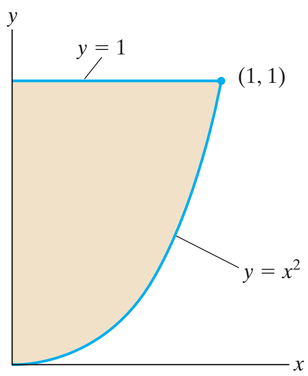
Solution: The height of the vertical strip is $1 - x^2$, so the area is $dA = (1 - x^2)\,dx$. Use this expression to evaluate Eq. (7.6).

The x coordinate of the centroid is

$$\bar{x} = \frac{\displaystyle\int_A x\,dA}{\displaystyle\int_A dA} = \frac{\displaystyle\int_0^1 x(1 - x^2)\,x}{\displaystyle\int_0^1 (1 - x^2)\,dA} = \frac{\left[\dfrac{x^2}{2} - \dfrac{x^4}{4}\right]_0^1}{\left[x - \dfrac{x^3}{3}\right]_0^1} = \frac{3}{8}$$

$$\boxed{\bar{x} = \frac{3}{8}}$$

Problem 7.3 In Example 7.2, suppose that the area is redefined as shown. Determine the y coordinate of the centroid.



Solution: The height of the vertical strip is $1 - x^2$, so the area is $dA = (1 - x^2)\,dx$.

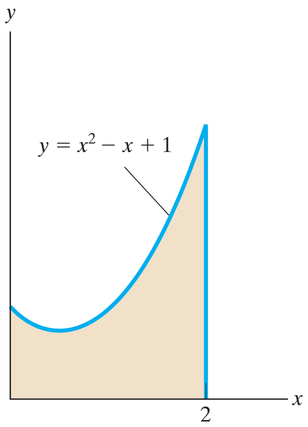
The y coordinate of the midpoint of the vertical strip is
$1 - \dfrac{1}{2}(1 - x^2) = \dfrac{1}{2}(1 + x^2)$.

We let this be the value of y in Eq. (7.7):

$$\bar{y} = \frac{\displaystyle\int_A y\,dA}{\displaystyle\int_A dA} = \frac{\displaystyle\int_0^1 \frac{1}{2}(1 + x^2)\,(1 - x^2)\,x}{\displaystyle\int_0^1 (1 - x^2)\,dA} = \frac{\left[\dfrac{x}{2} - \dfrac{x^5}{10}\right]_0^1}{\left[x - \dfrac{x^3}{3}\right]_0^1} = \frac{3}{5}$$

$$\boxed{\bar{y} = \frac{3}{5}}$$

Problem 7.4 Determine the centroid of the area.



Solution: The height of a vertical strip of width dx is $x^2 - x + 1$, so the area is $dA = (x^2 - x + 1)\ dx$. Use this expression to evaluate Eq. (7.6).

The x coordinate of the centroid is

$$\bar{x} = \frac{\int_A x dA}{\int_A dA} = \frac{\int_0^1 x(x^2 - x + 1)\ x}{\int_0^1 (x^2 - x + 1)\ dA} = \frac{\left[\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2}\right]_0^2}{\left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^2} = 1.25$$

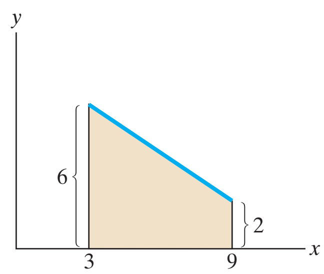
The y coordinate of the midpoint of the vertical strip is $\frac{1}{2}(x^2 - x + 1)$. We let this be the value of y in Eq. (7.7):

$$\bar{y} = \frac{\int_A y dA}{\int_A dA} = \frac{\int_0^1 \frac{1}{2}(x^2 - x + 1)^2\ x}{\int_0^1 (x^2 - x + 1)\ dA}$$

$$= \frac{\frac{1}{2}\left[\frac{x^5}{5} - \frac{2x^4}{4} + x^3 - x^2 + x\right]_0^2}{\left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^2} = 0.825$$

$$\boxed{\bar{x} = 1.25, \quad \bar{y} = 0.825}$$

Problem 7.5 Determine the coordinates of the centroid of the area.



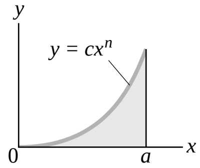
Solution: Use a vertical strip - The equation of the line is $y = 8 - \frac{2x}{3}$

$$\bar{x} = \frac{\int_3^9 x(y\,dx)}{\int_3^9 y\,dx} = \frac{\int_3^9 x\left(8 - \frac{2}{3}x\right)\,dx}{\int_3^9 \left(8 - \frac{2}{3}x\right)\,dx} = \frac{11}{2}$$

$$\bar{y} = \frac{\int_3^9 \frac{1}{2}y(y\,dx)}{\int_3^9 y\,dx} = \frac{\int_3^9 \frac{1}{2}\left(8 - \frac{2}{3}x\right)^2 dx}{\int_3^9 \left(8 - \frac{2}{3}x\right)\,dx} = \frac{13}{6}$$

$$\boxed{\begin{array}{l} \bar{x} = 5.5 \\ \bar{y} = 2.17 \end{array}}$$

Problem 7.6 Determine the x coordinate of the centroid of the area and compare your answers to the values given in Appendix B.



Solution:

$$A = \int_0^b \int_0^{cx^n} dy\,dx = \frac{cb^{n+1}}{n+1} \text{ provided that } n > -1$$

$$\boxed{\bar{x} = \frac{1}{A}\int_0^b \int_0^{cx^n} x\,dy\,dx = \frac{b(n+1)}{n+2}}$$

Matches the appendix

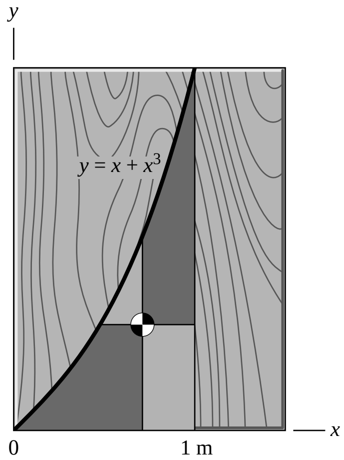
Problem 7.7 Determine the y coordinate of the centroid of the area and compare your answer to the value given in Appendix B.

Solution: See solution to 7.6

$$\bar{y} = \frac{1}{A}\int_0^b \int_0^{cx^n} y\,dy\,dx = \frac{b^n c(n+1)}{4n+2}$$

Matches the appendix

Problem 7.8 Suppose that an art student wants to paint a panel of wood as shown, with the horizontal and vertical lines passing through the centroid of the painted area, and asks you to determine the coordinates of the centroid. What are they?



Solution: *The area:*

$$A = \int_0^1 (x + x^3)\,dx = \left[\frac{x^2}{2} + \frac{x^4}{4}\right]_0^1 = \frac{3}{4}.$$

The x-coordinate:

$$\int_0^1 x(x + x^3)\,dx = \left[\frac{x^3}{3} + \frac{x^5}{5}\right]_0^1 = \frac{8}{15}.$$

Divide by the area: $\boxed{\mathbf{x} = \frac{32}{45} = 0.711}$

The y-coordinate: The element of area is $dA = (1 - x)\,dy$. Note that $dy = (1 + 3x^2)\,dx$, hence $dA = (1 - x)(1 + 3x^2)\,dx$. Thus

$$\mathbf{y}A = \int_A y\,dA = \int_0^1 (x + x^3)(1 - x)(1 + 3x^2)\,dx,$$

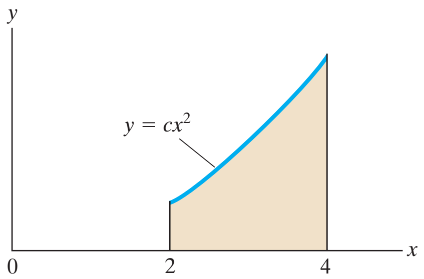
from which

$$\int_0^1 (x - x^2 + 4x^3 - 4x^4 + 3x^5 - 3x^6)\,dx$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{4}{4} - \frac{4}{5} + \frac{3}{6} - \frac{3}{7} = 0.4381.$$

Divide by A $\boxed{\mathbf{y} = 0.5841}$

Problem 7.9 Determine the value of the constant c so that the y coordinate of the centroid of the area is $\overline{y} = 2$. What is the x coordinate of the centroid?



Solution: The height of a vertical strip of width dx is cx^2 so the area is $dA = cx^2\,dx$.

The y coordinate of the midpoint of the vertical strip is $\frac{1}{2}cx^2$. We let this be the value of y in Eq. (7.7):

$$\overline{y} = \frac{\int_A y\,dA}{\int_A dA} = \frac{\int_2^4 \left(\frac{1}{2}cx^2\right) cx^2\,dx}{\int_2^4 cx^2\,dx} = \frac{\frac{c}{2}\left[\frac{x^5}{5}\right]_2^4}{\left[\frac{x^3}{3}\right]_2^4} = 5.31c$$

$$= 2 \Rightarrow c = 0.376$$

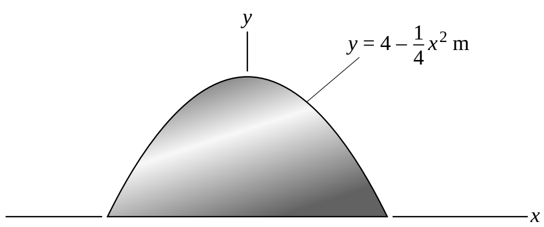
The x coordinate of the centroid is

$$\overline{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_2^4 x(cx^2)\,dx}{\int_2^4 cx^2\,dx} = \frac{\left[\frac{x^4}{4}\right]_2^4}{\left[\frac{x^3}{3}\right]_2^4} = 3.21$$

Notice that the value of $\overline{x}$ does not depend on the value of c.

$$\boxed{c = 0.376, \quad \overline{x} = 3.21}$$

Problem 7.10 Determine the coordinates of the centroid of the metal plate's cross-sectional area.



Solution: Let dA be a vertical strip:

The area $dA = y\,dx = \left(4 - \frac{1}{4}x^2\right) dx$. The curve intersects the x axis where $4 - \frac{1}{4}x^2 = 0$, or $x = \pm 4$.

Therefore

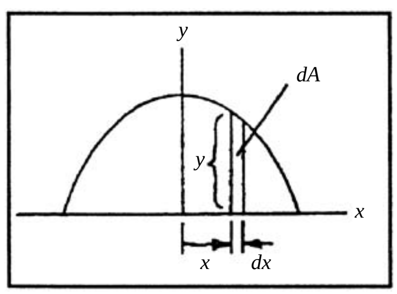
$$\mathbf{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_{-4}^4 \left(4x - \frac{1}{4}x^3\right) dx}{\int_{-4}^4 \left(4 - \frac{1}{4}x^2\right) dx} = \frac{\left[2x^2 - \frac{x^4}{16}\right]_{-4}^4}{\left[4x - \frac{x^3}{12}\right]_{-4}^4} = 0.$$

To determine $\mathbf{y}$, let y in equation (7.7) be the height of the midpoint of the vertical strip:

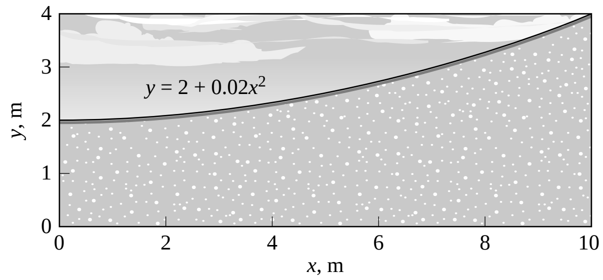
$$\mathbf{y} = \frac{\int_A y\,dA}{\int_A dA} = \frac{\int_{-4}^4 \frac{1}{2}\left(4 - \frac{1}{4}x^2\right)\left[\left(4 - \frac{1}{4}x^2\right) dx\right]}{\int_{-4}^4 \left(4 - \frac{1}{4}x^2\right) dx}$$

$$= \frac{\int_{-4}^4 \left(8 - x^2 + \frac{1}{32}x^4\right) dx}{\int_{-4}^4 \left(4 - \frac{x^2}{4}\right) dx} = \frac{\left[8x - \frac{x^3}{3} + \frac{x^5}{5(32)}\right]_{-4}^4}{\left[4x - \frac{x^3}{12}\right]_{-4}^4}$$

$$= \frac{34.1}{21.3} = 1.6 \text{ m}.$$



Problem 7.11 An architect wants to build a wall with the profile shown. To estimate the effects of wind loads, he must determine the wall's area and the coordinates of its centroid. What are they?

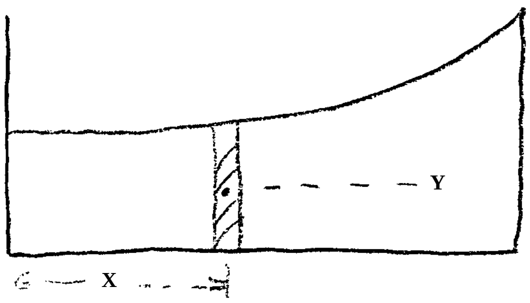


Solution:

$$\text{Area} = \int_0^{10} y\,dx = \int_0^{10} (2 + 0.02x^2)\,dx$$

$$\text{Area} = \left[2x + 0.02\frac{x^3}{3}\right]_0^{10} = 26.67\ \text{m}^2$$

$$dA = y\,dx = (2 + 0.02x^2)\,dx$$



$$\mathbf{x} = \frac{\int_0^{10} x\,dA}{\int_0^{10} dA} = \frac{\int_0^{10} (2x + 0.02x^3)\,dx}{26.67}$$

$$\mathbf{x} = \frac{\left(\cancel{2}\frac{x^2}{\cancel{2}} + 0.02\frac{x^4}{4}\right)\Big|_0^{10}}{26.67}\ \text{m}$$

$$\mathbf{x} = \frac{\left(100 + (0.02)\frac{10^4}{4}\right)}{26.67} = \frac{150}{26.67}$$

$$\mathbf{x} = 5.62\ \text{m}$$

$$\mathbf{y} = \frac{\int_0^{10} \left(\frac{y}{2}\right) y\,dx}{\int_0^{10} dA} = \frac{1}{(2)}\frac{\int_0^{10} (2 + 0.02x^2)^2\,dx}{(26.67)}$$

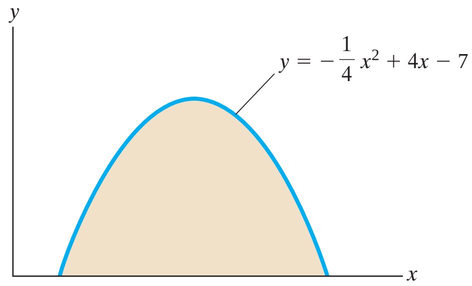
$$\mathbf{y} = \frac{1}{2(26.67)}\int_0^{10} (4 + 0.08x^2 + 0.0004x^4)\,dx$$

$$\mathbf{y} = \frac{\left(4x + 0.08\left(\frac{x^3}{3}\right) + 0.0004\left(\frac{x^5}{5}\right)\right)\Big|_0^{10}}{2(26.67)}$$

$$\mathbf{y} = \frac{74.67}{53.34}$$

$$\mathbf{y} = 1.40\ \text{m}$$

Problem 7.12 Determine the coordinates of the centroid of the area.



Solution: Use a vertical strip. We first need to find the x intercepts.

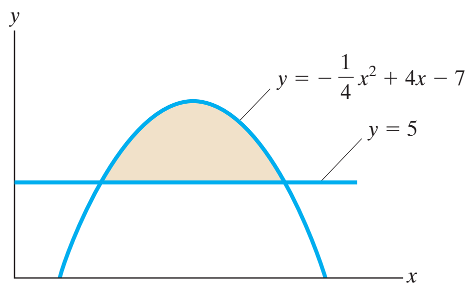
$$y = -\frac{1}{4}x^2 + 4x - 7 = 0 \;\Rightarrow\; x = 2, 14$$

$$\bar{x} = \frac{\displaystyle\int_2^{14} x(y\,dx)}{\displaystyle\int_2^{14} y\,dx} = \frac{\displaystyle\int_2^{14} x\left(-\frac{1}{4}x^2 + 4x - 7\right)dx}{\displaystyle\int_2^{14}\left(-\frac{1}{4}x^2 + 4x - 7\right)dx} = 8$$

$$\bar{y} = \frac{\displaystyle\int_2^{14} \frac{1}{2}y(y\,dx)}{\displaystyle\int_2^{14} y\,dx} = \frac{\displaystyle\int_2^{14} \frac{1}{2}\left(-\frac{1}{4}x^2 + 4x - 7\right)^2 dx}{\displaystyle\int_2^{14}\left(-\frac{1}{4}x^2 + 4x - 7\right)dx} = \frac{18}{5}$$

$$\boxed{\begin{array}{l}\bar{x} = 8\\ \bar{y} = 3.6\end{array}}$$

Problem 7.13 Determine the coordinates of the centroid of the area.



Solution: Use a vertical strip. We first need to find the x intercepts.

$$y = -\frac{1}{4}x^2 + 4x - 7 = 5 \;\Rightarrow\; x = 4, 12$$

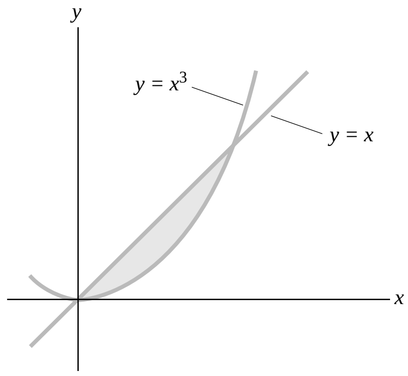
$$\bar{x} = \frac{\displaystyle\int_4^{12} x(y\,dx)}{\displaystyle\int_4^{12} y\,dx} = \frac{\displaystyle\int_4^{12} x\left[\left(-\frac{1}{4}x^2 + 4x - 7\right) - 5\right]dx}{\displaystyle\int_4^{12}\left[\left(-\frac{1}{4}x^2 + 4x - 7\right) - 5\right]dx} = 8$$

$$\bar{y} = \frac{\displaystyle\int_4^{12} y_c(y\,dx)}{\displaystyle\int_4^{12} y\,dx}$$

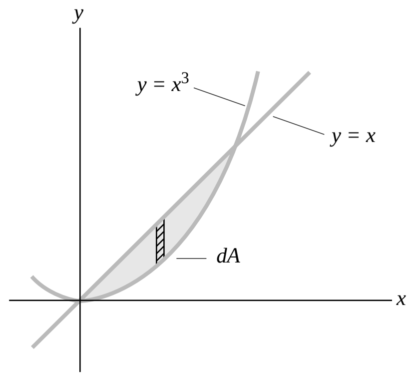
$$= \frac{\displaystyle\int_4^{12} \frac{1}{2}\left[\left(-\frac{1}{4}x^2 + 4x - 7\right) + 5\right]\left[\left(-\frac{1}{4}x^2 + 4x - 7\right) - 5\right]dx}{\displaystyle\int_4^{12}\left[\left(-\frac{1}{4}x^2 + 4x - 7\right) - 5\right]dx} = \frac{33}{5}$$

$$\boxed{\bar{x} = 8,\ \bar{y} = 6.6}$$

Problem 7.14 Determine the x coordinate of the centroid of the area.



Solution: Work this problem like Example 7.2



$$\mathbf{x} = \frac{\int_0^1 x\,dA}{\int_0^1 dA} = \frac{\int_0^1 x(x - x^3)\,dx}{\int_0^1 (x - x^3)\,dx}$$

$$\mathbf{x} = \frac{\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1}{\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1} = \frac{\left(\frac{1}{3} - \frac{1}{5}\right)}{\left(\frac{1}{2} - \frac{1}{4}\right)} = \frac{\frac{2}{15}}{\frac{1}{4}} = 0.533$$

$\mathbf{x} = 0.533$

Problem 7.15 Determine the y coordinate of the centroid of the area shown in Problem 7.14.

Solution: Solve this problem like example 7.2.

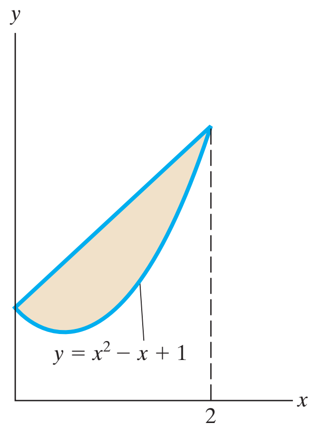
$$\mathbf{y} = \frac{\int_A y\,dA}{\int_A dA} = \frac{\int_0^1 \left[\frac{1}{2}(x + x^3)\right](x - x^3)\,dx}{\int_0^1 (x - x^3)\,dx}$$

$$\mathbf{y} = \frac{1}{2}\frac{\int_0^1 (x^2 - x^6)\,dx}{\int_0^1 (x - x^3)\,dx} = \frac{\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1}{2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1}$$

$$\mathbf{y} = \frac{\left[\frac{1}{3} - \frac{1}{7}\right]}{2\left[\frac{1}{2} - \frac{1}{4}\right]} = \frac{\left(\frac{4}{21}\right)}{2} = \frac{8}{21} = 0.381$$

$\mathbf{y} = 0.381$

Problem 7.16 Determine the x component of the centroid of the area.

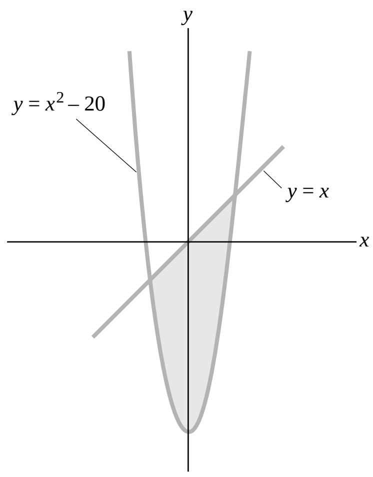


Solution: The value of the function $y = x^2 - x + 1$ at $x = 0$ is $y = 1$, and its value at $x = 2$ is $y = 3$. We need a function describing a straight line that passes through those points. Let $y = ax + b$. Determining the constants a and b from the conditions that $y = 1$ when $x = 0$ and $y = 3$ when $x = 2$, we obtain $a = 1$ and $b = 1$. The straight line is described by the function $y = x + 1$.

The height of the vertical strip of width dx is $(x + 1) - (x^2 - x + 1) = 2x - x^2$, so the area is $dA = (2x - x^3)\,dx$. Using this expression to evaluate Eq. (7.6).

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^2 x(2x - x^3)\,dx}{\int_0^2 (2x - x^3)\,dx} = \frac{\left[\frac{2x^3}{3} - \frac{x^5}{5}\right]_0^2}{\left[\frac{2x^2}{2} - \frac{x^4}{4}\right]_0^2} = 1 \qquad \boxed{\bar{x} = 1.}$$

Problem 7.17 Determine the x coordinate of the centroid of the area.



Solution: The intercept of the straight line with the parabola occurs at the roots of the simultaneous equations: $y = x$, and $y = x^2 - 20$. This is equivalent to the solution of the quadratic $x^2 - x - 20 = 0$, $x_1 = -4$, and $x_2 = 5$. These establish the limits on the integration. *The area:* Choose a vertical strip dx wide. The length of the strip is $(x - x^2 + 20)$, which is the distance between the straight line $y = x$ and the parabola $y = x^2 - 20$. Thus the element of area is $dA = (x - x^2 + 20)\,dx$ and

$$A = \int_{-4}^{+5} (x - x^2 + 20)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3} + 20x\right]_{-4}^{+5} = 121.5.$$

The x-coordinate:

$$\mathbf{x}A = \int_A x\,dA = \int_{-4}^{+5} (x^2 - x^3 + 20x)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} + 10x^2\right]_{-4}^{+5} = 60.75.$$

$$\boxed{\mathbf{x} = \frac{60.75}{121.5} = 0.5}$$

Problem 7.18 Determine the y coordinate of the centroid of the area in Problem 7.17.

Solution: Use the results of the solution to Problem 7.17 in the following.

The y-coordinate: The centroid of the area element occurs at the midpoint of the strip enclosed by the parabola and the straight line, and the y-coordinate is:

$$\mathbf{y} = x - \left(\frac{1}{2}\right)(x - x^2 + 20) = \left(\frac{1}{2}\right)(x + x^2 - 20).$$

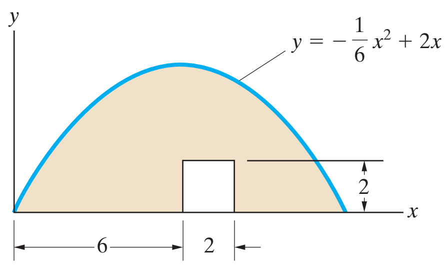
$$\mathbf{y}A = \int_A y\,dA = \left(\frac{1}{2}\right)\int_{-4}^{5}(x + x^2 - 20)(x - x^2 + 20)\,dx$$

$$= \left(\frac{1}{2}\right)\int_{-4}^{+5}(-x^4 + 41x^2 - 400)\,dx$$

$$= \left(\frac{1}{2}\right)\left[-\frac{x^5}{5} + \frac{41x^3}{3} - 400x\right]_{-4}^{5} = -923.4.$$

$$\boxed{\mathbf{y} = -\frac{923.4}{121.5} = -7.6}$$

Problem 7.19 What is the x coordinate of the centroid of the area?



Solution: Use vertical strips, do an integral for the parabola then subtract the square

First find the intercepts

$$y = -\frac{1}{6}x^2 + 2x = 0 \Rightarrow x = 0, 12$$

$$\bar{x} = \frac{\displaystyle\int_0^{12} x\left(-\frac{1}{6}x^2 + 2x\right)dx - 7(2)(2)}{\displaystyle\int_0^{12}\left(-\frac{1}{6}x^2 + 2x\right)dx - (2)(2)} = \frac{65}{11}$$

$$\boxed{\bar{x} = 5.91}$$

Problem 7.20 What is the y coordinate of the centroid of the area in Problem 7.19?

Solution: Use vertical strips, do an integral for the parabola then subtract the square

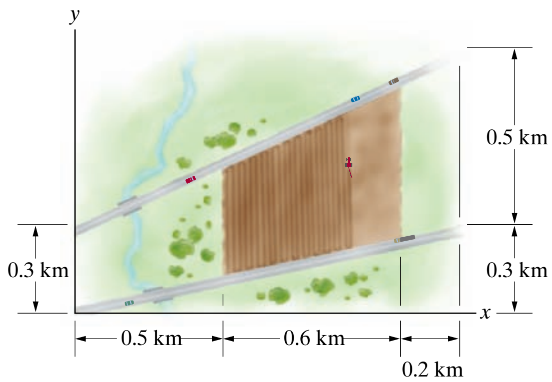
First find the intercepts

$$y = -\frac{1}{6}x^2 + 2x = 0 \Rightarrow x = 0, 12$$

$$\bar{y} = \frac{\displaystyle\int_0^{12} \frac{1}{2}\left(-\frac{1}{6}x^2 + 2x\right)^2 dx - 1(2)(2)}{\displaystyle\int_0^{12}\left(-\frac{1}{6}x^2 + 2x\right)dx - (2)(2)} = \frac{139}{55}$$

$$\boxed{\bar{y} = 2.53}$$

Problem 7.21 An agronomist wants to measure the rainfall at the centroid of a plowed field between two roads. What are the coordinates of the point where the rain gauge should be placed?



Solution: *The area:* The element of area is the vertical strip $(y_t - y_b)$ long and dx wide, where $y_t = m_t x + b_t$ and $y_b = m_b x + b_b$ are the two straight lines bounding the area, where

$$m_t = \frac{(0.8 - 0.3)}{(1.3 - 0)} = 0.3846,$$

and $b_t = 0.8 - 1.3\ \mathrm{m}_t = 0.3$.

Similarly:

$$m_b = \frac{(0.3 - 0)}{(1.3 - 0)} = 0.2308,$$

and $b_b = 0$.

The element of area is

$$dA = (y_t - y_b)\,dx = ((m_t - m_b)x + b_t - b_b)\,dx$$

$$= (0.1538x + 0.3)\,dx,$$

from which

$$A = \int_{0.5}^{1.1} (0.1538x + 0.3)\,dx$$

$$= \left[0.1538\frac{x^2}{2} + 0.3x\right]_{0.5}^{1.1} = 0.2538 \text{ sq km}.$$

The x-coordinate:

$$\int_A x\,dA = \int_{0.5}^{1.1} (0.1538x + 0.3)x\,dx$$

$$= \left[0.1538\frac{x^3}{3} + 0.3\frac{x^2}{2}\right]_{0.5}^{1.1} = 0.2058.$$

$$\boxed{\mathbf{x} = 0.8109 \text{ km}}$$

The y-coordinate: The y-coordinate of the centroid of the elemental area is

$$\mathbf{y} = y_b + (\tfrac{1}{2})(y_t - y_b) = (\tfrac{1}{2})(y_t + y_b) = 0.3077x + 0.15.$$

Thus, $\mathbf{y}A = \int_A y\,dA$

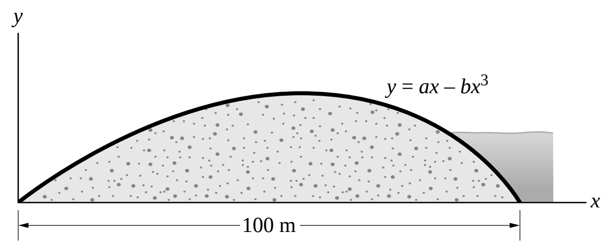
$$= \int_{0.5}^{1.1} (0.3077x + 0.15)(0.1538x + 0.3)\,dx$$

$$= \int_{0.5}^{1.1} (0.0473x^2 + 0.1154x + 0.045)\,dx$$

$$= \left[0.0471\frac{x^3}{3} + 0.1153\frac{x^2}{2} + 0.045x\right]_{0.5}^{1.1} = 0.1014.$$

Divide by the area: $\boxed{\mathbf{y} = \frac{0.1014}{0.2538} = 0.3995 \text{ km}}$

Problem 7.22 The cross section of an earth-fill dam is shown. Determine the coefficients a and b so that the y coordinate of the centroid of the cross section is 10 m.



Solution: *The area:* The elemental area is a vertical strip of length y and width dx, where $y = ax - bx^3$. Note that $y = 0$ at $x = 100$, thus $b = a \times 10^{-4}$. Thus

$$A = \int_A dA = a\int_0^{100} (x - (10^{-4})x^3)\,dx$$

$$= (0.5a)[x^2 - (0.5 \times 10^{-4})x^4]_0^{100}$$

$$= 0.5a \times 10^4 - 0.25b \times 10^8,$$

and the area is $A = 0.25a \times 10^4$. *The y-coordinate:* The y-coordinate of the centroid of the elemental area is

$$\mathbf{y} = (0.5)(ax - bx^3) = (0.5a)(x - (10^{-4})x^3),$$

from which

$$\mathbf{y}A = \int_A y\,dA$$

$$= (0.5)a^2 \int_0^{100} (x - (10^{-4})x^3)^2\,dx$$

$$= (0.5)a^2 \int_0^{100} (x^2 - 2(10^{-4})x^4 + (10^{-8})x^6)\,dx$$

$$= (0.5a^2)\left[\frac{x^3}{3} - (10^{-4})\frac{2x^5}{5} + (10^{-8})\frac{x^7}{7}\right]_0^{100}$$

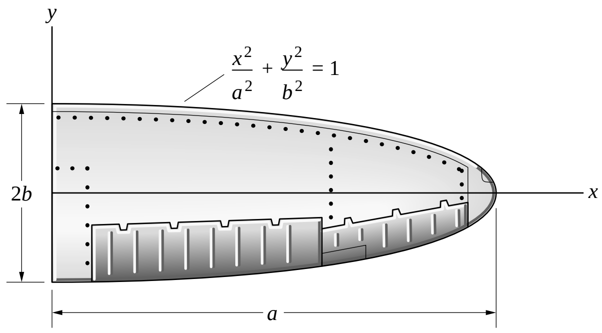
$$= 3.81a^2 \times 10^4.$$

Divide by the area:

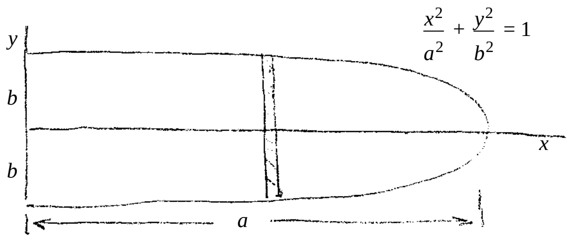
$$\mathbf{y} = \frac{3.810a^2 \times 10^4}{0.25a \times 10^4} = 15.2381a.$$

For $\mathbf{y} = 10$, $\boxed{a = 0.6562}$, and $\boxed{b = 6.562 \times 10^{-5}\text{m}^{-2}}$

Problem 7.23 The Supermarine Spitfire used by Great Britain in World War II had a wing with an elliptical profile. Determine the coordinates of its centroid.



Solution:

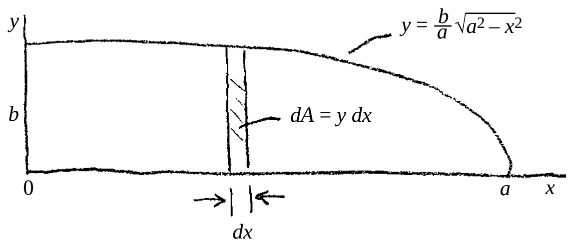


By symmetry, $\mathbf{y} = 0$.

From the equation of the ellipse,

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

By symmetry, the **x** centroid of the wing is the same as the **x** centroid of the upper half of the wing. Thus, we can avoid dealing with $\pm$ values for y.



$$\mathbf{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{b}{a}\int_0^a x\sqrt{a^2 - x^2}\,dx}{\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx}$$

Using integral tables

$$\int x\sqrt{a^2 - x^2}\,dx = -\frac{(a^2 - x^2)^{3/2}}{3}$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

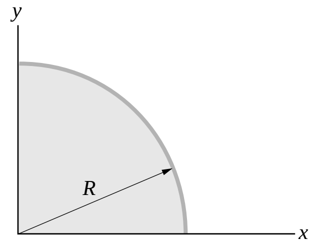
Substituting, we get

$$\mathbf{x} = \frac{\left[-\left(a^2 - x^2\right)^{3/2}/3\right]_0^a}{\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a}$$

$$\mathbf{x} = \frac{\left[-0 + a^3/3\right]}{\left[0 + \frac{a^2}{2}\left(\frac{\pi}{2}\right) - 0 - 0\right]} = \frac{a^3/3}{a^2\pi/4}$$

$$\mathbf{x} = \frac{4a}{3\pi}$$

Problem 7.24 Determine the coordinates of the centroid of the area.



Strategy: Write the equation for the circular boundary in the form $y = (R^2 - x^2)^{1/2}$ and use a vertical "strip" of width dx as the element of area dA.

Solution: *The area:* The equation of the circle is $x^2 + y^2 = R^2$. Take the elemental area to be a vertical strip of height $y = \sqrt{R^2 - x^2}$ and width dx, hence the element of area is $dA = \sqrt{R^2 - x^2}\,dx$. The area is $A = \frac{A_{circle}}{4} = \frac{\pi R^2}{4}$. *The x-coordinate:*

$$\mathbf{x}A = \int_A x\,dA = \int_0^R x\sqrt{R^2 - x^2}\,dx = \left[-\frac{(R^2 - x^2)^{3/2}}{3}\right]_0^R = \frac{R^3}{3}:$$

$$\boxed{\mathbf{x} = \frac{4R}{3\pi}}$$

The y-coordinate: The y-coordinate of the centroid of the element of area is at the midpoint:

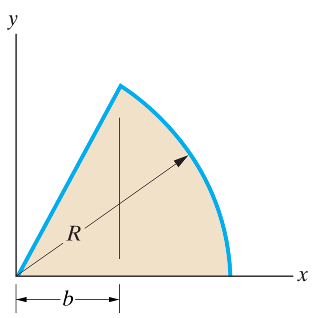
$$\mathbf{y} = (\tfrac{1}{2})\sqrt{R^2 - x^2},$$

$$\text{hence } \mathbf{y}A = \int_A y\,dA = \left(\frac{1}{2}\right)\int_0^R (R^2 - x^2)\,dx$$

$$= \left(\frac{1}{2}\right)\left[R^2 x - \frac{x^3}{3}\right]_0^R = \frac{R^3}{3}$$

$$\boxed{\mathbf{y} = \frac{4R}{3\pi}}$$

Problem 7.25* If $R = 6$ and $b = 3$, what is the y coordinate of the centroid of the area?

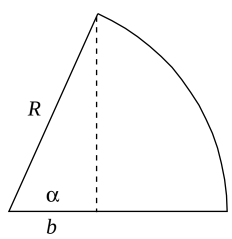


Solution: We will use polar coordinates. First find the angle α

$$\alpha = \cos^{-1}\left(\frac{b}{R}\right) = \cos^{-1}\left(\frac{3}{6}\right) = 60^\circ = \frac{\pi}{3}$$

$$A = \int_0^\alpha \int_0^R r\,dr\,d\theta = \int_0^{\pi/3}\int_0^6 r\,dr\,d\theta = 6\pi$$

$$\boxed{\overline{y} = \frac{1}{A}\int_0^{\pi/3}\int_0^6 r^2 \sin\theta\,dr\,d\theta = \frac{6}{\pi} = 1.910}$$

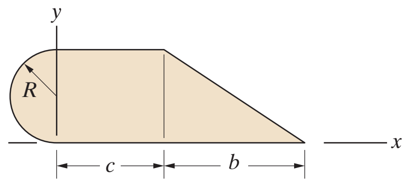


Problem 7.26* What is the x coordinate of the centroid of the area in Problem 7.25?

Solution: See the solution to 7.25

$$\boxed{\overline{x} = \frac{1}{A}\int_0^{\pi/3}\int_0^6 r^2 \cos\theta\,dr\,d\theta = \frac{6\sqrt{3}}{\pi} = 3.31}$$

Problem 7.27 In Active Example 7.3, suppose that the area is placed as shown. Let the dimensions $R = 6$ cm, $c = 14$ cm, and $b = 18$ cm. Use Eq. (7.9) to determine the x coordinate of the centroid.



Solution: Let the semicircular area be area 1, let the rectangular area be area 2, and let the triangular area be area 3. The areas and the x coordinates of their centroids are

$$A_1 = \frac{1}{2}\pi R^2, \qquad \bar{x}_1 = -\frac{4R}{3\pi},$$

$$A_2 = c\,(2R), \qquad \bar{x}_2 = \frac{1}{2}\,c,$$

$$A_3 = \frac{1}{2}\,b\,(2R), \qquad \bar{x}_3 = c + \frac{1}{3}\,b$$

The x coordinate of the centroid of the composite area is

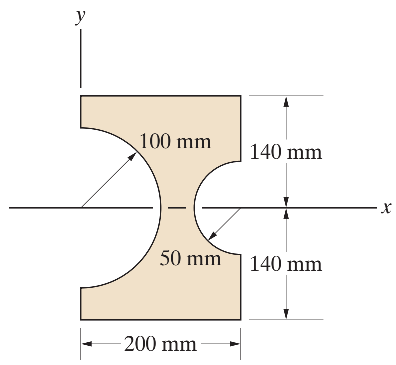
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{\left(-\frac{4R}{3\pi}\right)\left(\frac{1}{2}\pi R^2\right) + \left(\frac{1}{2}c\right)(2cR) + \left(c + \frac{1}{3}b\right)(bR)}{\frac{1}{2}\pi R^2 + 2cR + bR}$$

Substituting the values for R, b, and c yields

$$\boxed{\bar{x} = 9.60 \text{ cm}}$$

Problem 7.28 In Example 7.4, suppose that the area is given a second semicircular cutout as shown. Determine the x coordinate of the centroid.



Solution: Let the rectangular area *without the cutouts* be area 1, let the left cutout be area 2, and let the right cutout be area 3. The areas and the x coordinates of their centroids are

$$A_1 = (200)\,(280)\text{ mm}^2, \qquad \bar{x}_1 = 100 \text{ mm},$$

$$A_2 = -\frac{1}{2}\,\pi(100)^2\text{ mm}^2, \qquad \bar{x}_2 = \frac{4(100)}{3\pi}\text{ mm},$$

$$A_3 = -\frac{1}{2}\,\pi(50)^2\text{ mm}^2, \qquad \bar{x}_3 = 200 - \frac{4(50)}{3\pi}\text{ mm}.$$

The x coordinated of the centroid of the composite area is

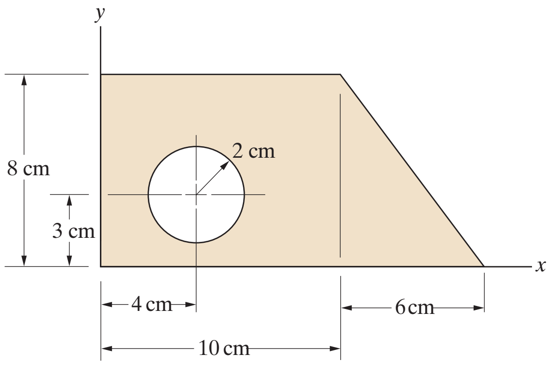
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(100)\,[(200)\,(280)] + \left[\frac{4(100)}{3\pi}\right]\left[-\frac{1}{2}\,\pi\,(100)^2\right] + \left[200 - \frac{4(50)}{3\pi}\right]\left[-\frac{1}{2}\,\pi(50)^2\right]}{(200)\,(280) - \frac{1}{2}\,\pi(100)^2 - \frac{1}{2}\,\pi(50)^2}$$

$$= 116 \text{ mm}.$$

$$\boxed{\bar{x} = 116 \text{ mm}}$$

Problem 7.29 Determine the coordinates of the centroids.



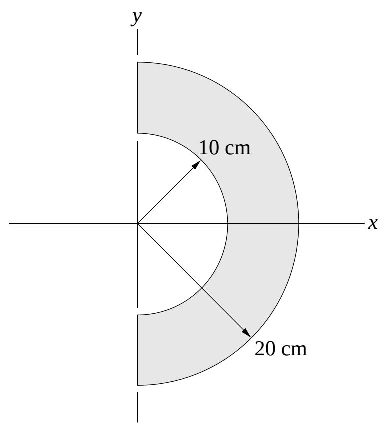
Solution: Break into a rectangle, a triangle and a circular hole

$$\bar{x} = \frac{5[(10)(8)] + 12\left[\frac{1}{2}(8)(6)\right] - 4[\pi(2)^2]}{(10)(8) + \frac{1}{2}(8)(6) - \pi(2)^2} = 6.97 \text{ cm}$$

$$\bar{y} = \frac{4[(10)(8)] + \frac{1}{3}8\left[\frac{1}{2}(8)(6)\right] - 3[\pi(2)^2]}{(10)(8) + \frac{1}{2}(8)(6) - \pi(2)^2} = 3.79 \text{ cm}$$

$$\boxed{\begin{array}{l} \bar{x} = 6.97 \text{ cm} \\ \bar{y} = 3.79 \text{ cm} \end{array}}$$

Problem 7.30 Determine the coordinates of the centroids.



Solution: The strategy is to find the centroid for the half circle area, and use the result in the composite algorithm. *The area:* The element of area is a vertical strip y high and dx wide. From the equation of the circle, $y = \pm\sqrt{R^2 - x^2}$. The height of the strip will be twice the positive value, so that $dA = 2\sqrt{R^2 - x^2}\,dx$, from which

$$A = \int_A dA = 2\int_0^R (R^2 - x^2)^{1/2}\,dx$$

$$= 2\left[\frac{x\sqrt{R^2 - x^2}}{2} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right]_0^R = \frac{\pi R^2}{2}$$

The x-coordinate:

$$\int_A x\,dA = 2\int_0^R x\sqrt{R^2 - x^2}\,dx$$

$$= 2\left[-\frac{(R^2 - x^2)^{3/2}}{3}\right]_0^R = \frac{2R^3}{3}.$$

Divide by A: $\mathbf{x} = \dfrac{4R}{3\pi}$

The y-coordinate: From symmetry, the y-coordinate is zero.

The composite: For a complete half circle $\mathbf{x}_1 = \dfrac{4(20)}{3\pi} = 8.488$ cm. For the inner half circle $\mathbf{x}_2 = 4.244$ cm. The areas are

$A_1 = 628.32 \text{ cm}^2$

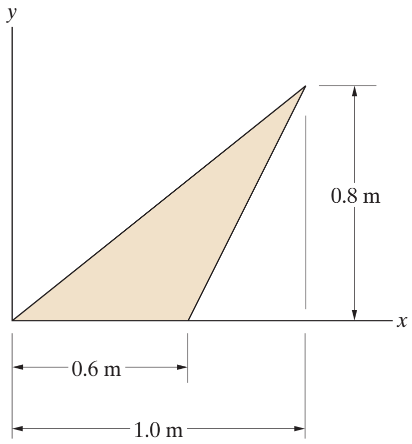
and $A_2 = 157.08 \text{ cm}^2$.

$$\bar{x} = \frac{8.488 \times 628.32 - 4.244 \times 157.08}{628.31 - 157.08}$$

$= 9.90$ cm

$\bar{y} = 0$

Problem 7.31 Determine the coordinates of the centroids.



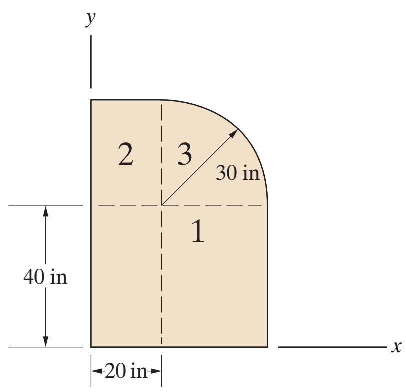
Solution: Use a big triangle and a triangular hole

$$\bar{x} = \frac{\frac{2}{3}1.0\left[\frac{1}{2}(1.0)(0.8)\right] - \left(0.6 + \frac{2}{3}0.4\right)\left[\frac{1}{2}(0.4)(0.8)\right]}{\frac{1}{2}(1.0)(0.8) - \frac{1}{2}(0.4)(0.8)} = 0.533$$

$$\bar{y} = \frac{\frac{1}{3}0.8\left[\frac{1}{2}(1.0)(0.8)\right] - \left(\frac{1}{3}0.8\right)\left[\frac{1}{2}(0.4)(0.8)\right]}{\frac{1}{2}(1.0)(0.8) - \frac{1}{2}(0.4)(0.8)} = 0.267$$

$$\boxed{\begin{array}{l}\bar{x} = 0.533 \text{ m} \\ \bar{y} = 0.267 \text{ m}\end{array}}$$

Problem 7.32 Determine the coordinates of the centroid.



Solution: Let the area be divided into parts as shown. The areas and the coordinates are

$A_1 = (40)\ (50)\ \text{in}^2, \quad \bar{x}_1 = 25 \text{ in}, \quad \bar{y}_1 = 20 \text{ in},$

$A_2 = (20)\ (30)\ \text{in}^2, \quad \bar{x}_2 = 10 \text{ in}, \quad \bar{y}_2 = 40 + 15 \text{ in},$

$A_3 = \frac{1}{4}\pi(30)^2\ \text{in}^2, \quad \bar{x}_3 = 20 + \frac{4(30)}{3\pi}\text{in.} \quad \bar{y}_3 = 40 + \frac{4(30)}{3\pi}\text{in.}$

The x coordinate of the centroid of the composite area is

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{25\,[(40)\ (50)] + [10]\,[(20)\ (30)] + \left[20 + \frac{4(30)}{3\pi}\right]\left[\frac{1}{4}\pi(30)^2\right]}{(40)\ (50) + (20)\ (30) + \frac{1}{4}\pi(30)^2}$$

$$= 23.9 \text{ in.}$$

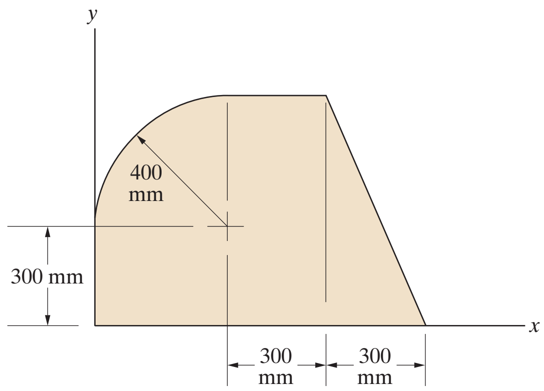
The y coordinate of the centroid of the composite area is

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(20)\,[(40)\ (50)] + [55]\,[(20)\ (30)] + \left[40 + \frac{4(30)}{3\pi}\right]\left[\frac{1}{4}\pi(30)^2\right]}{(40)\ (50) + (20)\ (30) + \frac{1}{4}\pi(30)^2} = 33.3 \text{ in.}$$

$\boxed{\bar{x} = 23.9 \text{ in}, \bar{y} = 33.3 \text{ in.}}$

Problem 7.33 Determine the coordinates of the centroids.



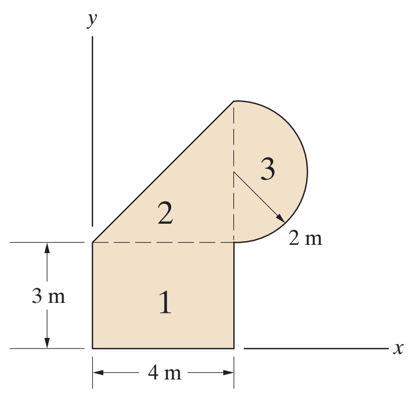
Solution: Break into 4 pieces (2 rectangles, a quarter circle, and a triangle)

$$\bar{x} = \frac{0.2[(0.4)(0.3)] + \left(0.4 - \dfrac{4[0.4]}{3\pi}\right)\left(\dfrac{\pi[0.4]^2}{4}\right) + 0.55(0.3)(0.7) + (0.8)\left[\dfrac{1}{2}(0.3)(0.7)\right]}{(0.4)(0.3) + \dfrac{\pi[0.4]^2}{4} + (0.3)(0.7) + \dfrac{1}{2}(0.3)(0.7)}$$

$$\bar{y} = \frac{0.15[(0.4)(0.3)] + \left(0.3 + \dfrac{4[0.4]}{3\pi}\right)\left(\dfrac{\pi[0.4]^2}{4}\right) + 0.35(0.3)(0.7) + \left(\dfrac{1}{3}0.7\right)\left[\dfrac{1}{2}(0.3)(0.7)\right]}{(0.4)(0.3) + \dfrac{\pi[0.4]^2}{4} + (0.3)(0.7) + \dfrac{1}{2}(0.3)(0.7)}$$

$\boxed{\bar{x} = 0.450 \text{ m}, \ \bar{y} = 0.312 \text{ m}}$

Problem 7.34 Determine the coordinates of the centroid.



Solution: Let the area be divided into parts as shown. The areas and the coordinates are

$A_1 = (4)\ (3)\ \text{m}^2, \quad \bar{x}_1 = 2 \text{ m}, \quad \bar{y}_1 = 1.5 \text{ m},$

$A_2 = \dfrac{1}{2}(4)\ (4)\ \text{m}^2, \quad \bar{x}_2 = \dfrac{2}{3}(4) \text{ m}, \quad \bar{y}_2 = 3 + \dfrac{1}{3}(4) \text{ m},$

$A_3 = \dfrac{1}{2}\pi\ (2)^2\ \text{m}^2, \quad \bar{x}_3 = 4 + \dfrac{4(2)}{3\pi} \text{ m}. \quad \bar{y}_3 = 3 + 2 \text{ m}.$

The x coordinate of the centroid of the composite area is

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(2)\ [(4)\ (3)] + \left[\dfrac{2}{3}\ (4)\right]\left[\dfrac{1}{2}\ (4)\ (4)\right] + \left[4 + \dfrac{4(2)}{3\pi}\right]\left[\dfrac{1}{2}\pi\ (2)^2\right]}{(4)\ (3) + \dfrac{1}{2}\ (4)\ (4) + \dfrac{1}{2}\pi\ (2)^2} = 2.88 \text{ m}.$$

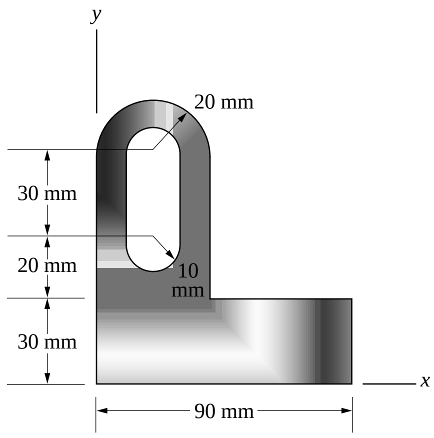
The y coordinate of the centroid of the composite area is

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

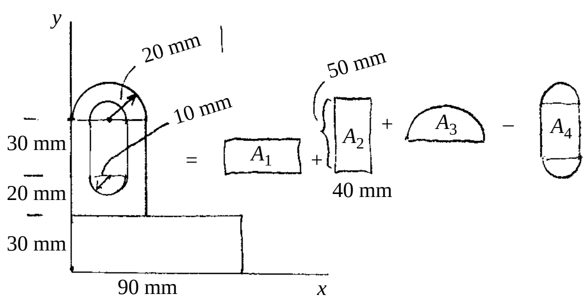
$$= \frac{(1.5)\ [(4)\ (3)] + \left[3 + \dfrac{1}{3}\ (4)\right]\left[\dfrac{1}{2}\ (4)\ (4)\right] + [3 + 2]\left[\dfrac{1}{2}\pi\ (2)^2\right]}{(4)\ (3) + \dfrac{1}{2}\ (4)\ (4) + \dfrac{1}{2}\pi\ (2)^2} = 3.20 \text{ m}.$$

$\boxed{\bar{x} = 2.88 \text{ m}, \quad \bar{y} = 3.20 \text{ m}.}$

Problem 7.35 Determine the coordinates of the centroids.



Solution: Determine this result by breaking the compound object into parts



A_1 : $A_1 = (30)(90) = 2700\ \text{mm}^2$

$\mathbf{x}_1 = 45$ mm

$\mathbf{y}_1 = 15$ mm

A_2: (sits on top of A_1)

$A_2 = (40)(50) = 2000\ \text{mm}^2$

$\mathbf{x}_2 = 20$ mm

$\mathbf{y}_2 = 30 + 25 = 55$ mm

A_3 : $A_3 = \frac{1}{2}\pi r_0^2 = \frac{\pi}{2}(20)^2 = 628.3\ \text{mm}^2$

$\mathbf{x}_3 = 20$ mm

$\mathbf{y}_3 = 80\ \text{mm} + \frac{4r_0}{3\pi} = 88.49$ mm

A_4 : $A_4 = (30)(20) + \pi r_i^2$

$A_4 = 600 + \pi(10)^2 = 914.2\ \text{mm}^2$

$\mathbf{x}_4 = 20$ mm

$\mathbf{y}_4 = 50 + 15 = 65$ mm

Area (composite)

$= A_1 + A_2 + A_3 - A_4$

$= 4414.2\ \text{mm}^2$

For the composite:

$$\mathbf{x} = \frac{\mathbf{x}_1 A_1 + \mathbf{x}_2 A_2 + \mathbf{x}_3 A_3 - \mathbf{x}_4 A_4}{(A_1 + A_2 + A_3 - A_4)}$$

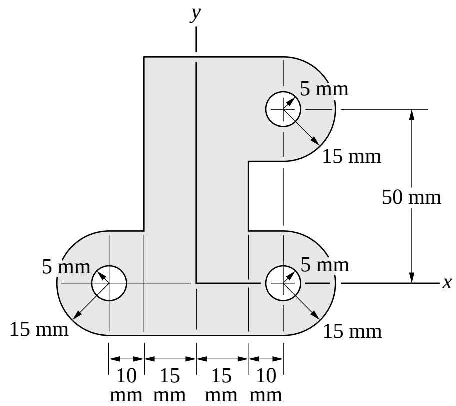
$$\mathbf{x} = \frac{155782}{4414.2} = 35.3\ \text{mm}$$

$$\mathbf{y} = \frac{\mathbf{y}_1 A_1 + \mathbf{y}_2 A_2 + \mathbf{y}_3 A_3 - \mathbf{y}_4 A_4}{A_1 + A_2 + A_3 - A_4}$$

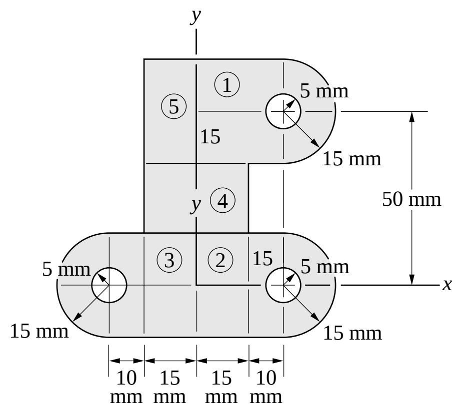
$$\mathbf{y} = \frac{146675}{4414.2} = 33.2\ \text{mm}$$

The value for **y** is not the same as in the new problem statement. This value seems correct. (The **x** value checks).

Problem 7.36 Determine the coordinates of the centroids.



Solution: Comparison of the solution to Problem 7.29 and our areas 1, 2, and 3, we see that in order to use the solution of Problem 7.29, we must set $a = 25$ mm, $R = 15$ mm, and $r = 5$ mm. If we do this, we find that for this shape, measuring from the y axis, $\mathbf{x} = 18.04$ mm. The corresponding areas for regions 1, 2, and 3 is $1025\ \text{mm}^2$. The centroids of the rectangular areas are at their geometric centers. By inspection, we how have the following information for the five areas



Area 1: $\text{Area}_1 = 1025\ \text{mm}^2$, $\mathbf{x}_1 = 18.04$ mm, and $\mathbf{y}_1 = 50$ mm.

Area 2: $\text{Area}_2 = 1025\ \text{mm}^2$, $\mathbf{x}_2 = 18.04$ mm, and $\mathbf{y}_2 = 0$ mm.

Area 3: $\text{Area}_3 = 1025\ \text{mm}^2$, $\mathbf{x}_3 = -18.04$ mm, and $\mathbf{y}_3 = 0$ mm.

Area 4: $\text{Area}_4 = 600\ \text{mm}^2$, $\mathbf{x}_4 = 0$ mm, and $\mathbf{y}_4 = 25$ mm.

Area 5: $\text{Area}_5 = 450\ \text{mm}^2$, $\mathbf{x}_5 = -7.5$ mm, and $\mathbf{y}_5 = 50$ mm.

Combining the properties of the five areas, we can calculate the centroid of the composite area made up of the five regions shown.

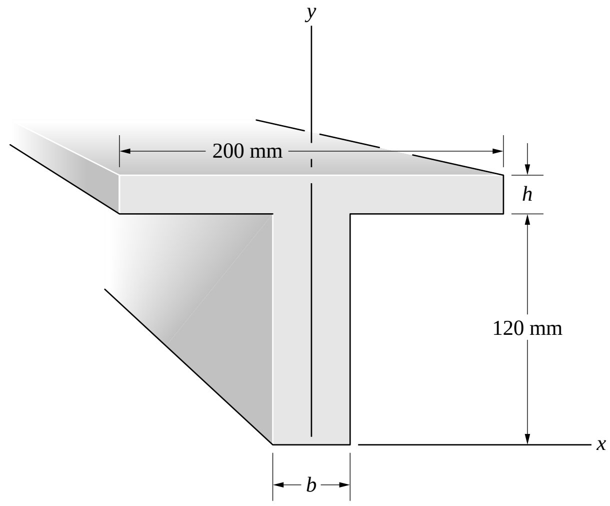
$$\text{Area}_{\text{TOTAL}} = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4 + \text{Area}_5$$

$$= 4125\ \text{mm}^2.$$

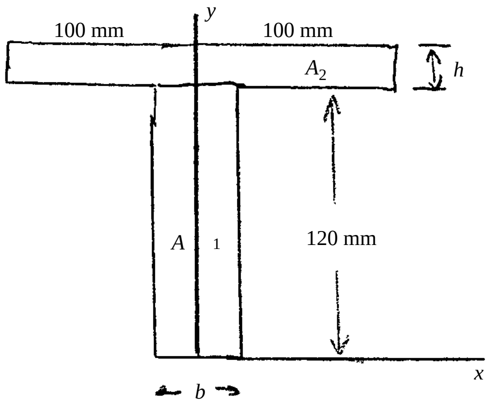
$$\text{Then,}\ \mathbf{x} = (\mathbf{x}_1\text{Area}_1 + \mathbf{x}_2\text{Area}_2 + \mathbf{x}_3\text{Area}_3 + \mathbf{x}_4\text{Area}_4 + \mathbf{x}_5\text{Area}_5)/\text{Area}_{\text{TOTAL}} = 3.67\ \text{mm},$$

$$\text{and}\ \mathbf{y} = (\mathbf{y}_1\text{Area}_1 + \mathbf{y}_2\text{Area}_2 + \mathbf{y}_3\text{Area}_3 + \mathbf{y}_4\text{Area}_4 + \mathbf{y}_5\text{Area}_5)/\text{Area}_{\text{TOTAL}} = 21.52\ \text{mm}.$$

Problem 7.37 The dimensions $b = 42$ mm and $h = 22$ mm. Determine the y coordinate of the centroid of the beam's cross section.



Solution: Work as a composite shape



$b = 42$ mm

$h = 22$ mm

$A_1 = 120\ b\ \text{mm}^2 = 5040\ \text{mm}^2$

$$\left.\begin{aligned}\mathbf{x}_1 &= 0\\ \mathbf{y}_1 &= 60\ \text{mm}\end{aligned}\right\}\ \text{by symmetry}$$

$A_2 = 200\ h = 4400\ \text{mm}^2$

$\mathbf{x}_2 = 0$

$$\mathbf{y}_2 = 120 + \frac{h}{2} = 131\ \text{mm}$$

$$\mathbf{x} = \frac{A_1\mathbf{x}_1 + A_2\mathbf{x}_2}{A_1 + A_2} = \frac{0+0}{9440}$$

$\mathbf{x} = 0$

$$\mathbf{y} = \frac{A_1\mathbf{y}_1 + A_2\mathbf{y}_2}{A_1 + A_2} = 93.1\ \text{mm}$$

Problem 7.38 If the cross-sectional area of the beam shown in Problem 7.37 is 8400 mm^2 and the y coordinate of the centroid of the area is $\overline{y} = 90$ mm, what are the dimensions b and h?

Solution: From the solution to Problem 7.37

$A_1 = 120\ b,\ A_2 = 200\ h$

and $\mathbf{y} = \dfrac{\mathbf{y}_1 A_1 + \mathbf{y}_2 A_2}{A_1 + A_2}$

$$\mathbf{y} = \frac{(60)(120\ b) + \left(120 + \dfrac{h}{2}\right)(200\ h)}{120\ b + 200\ h}$$

where $\mathbf{y}_1 = 60$ mm

$\mathbf{y} = 90$ mm

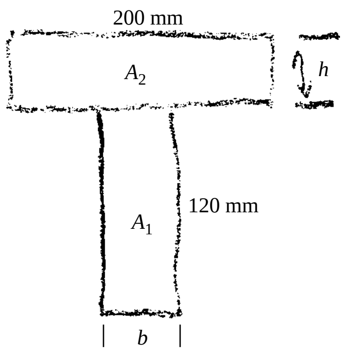
$A_1 + A_2 = 8400$ mm^2

Also, $\mathbf{y}_2 = 120 + h/2$

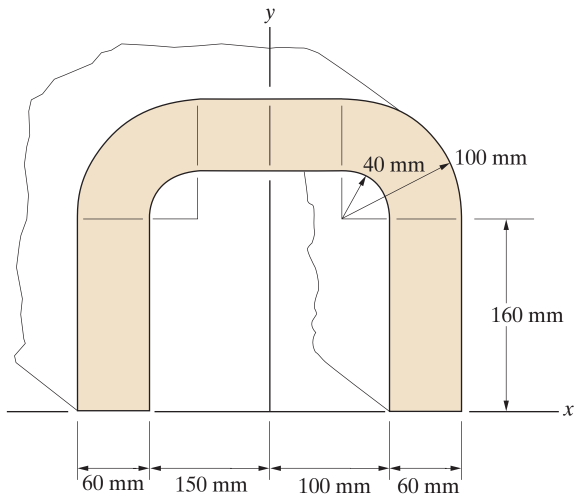
Solving these equations simultaneously we get

$h = 18.2$ mm

$b = 39.7$ mm



Problem 7.39 Determine the y coordinate of the centroid of the beam's cross section.

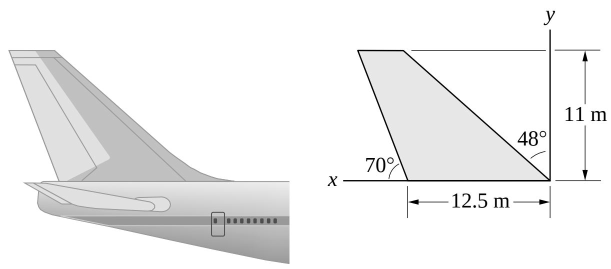


Solution: Take advantage of the symmetry. Work with only one half of the structure. Break into 2 rectangles, a quarter circle, and a quarter circular hole.

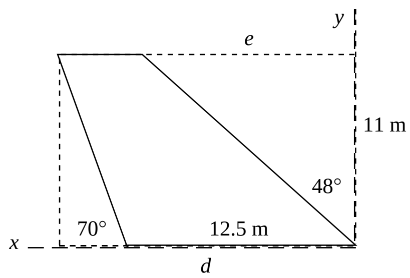
$$\overline{y} = \frac{0.08(0.06)(0.16) + (0.23)(0.06)(0.06) + \left(0.16 + \dfrac{4[0.1]}{3\pi}\right)\left(\dfrac{\pi[0.1]^2}{4}\right) - \left(0.16 + \dfrac{4[0.04]}{3\pi}\right)\left(\dfrac{\pi[0.04]^2}{4}\right)}{(0.06)(0.16) + (0.06)(0.06) + \dfrac{\pi[0.1]^2}{4} - \dfrac{\pi[0.04]^2}{4}}$$

$$\boxed{\overline{y} = 0.1497\,\text{m}}$$

Problem 7.40 Determine the coordinates of the centroid of the airplane's vertical stabilizer.



Solution: We work with a rectangle and two triangular holes



We have

$d = 12.5\text{ m} + 11\text{ m}\cot 70^\circ = 16.50\text{ m}$

$e = 11\text{ m}\tan 48^\circ = 12.22\text{ m}$

In the x direction

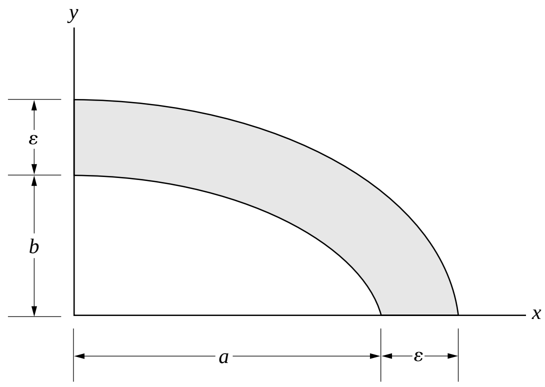
$$\overline{x} = \frac{\frac{1}{2}d[d(11)] - \frac{1}{3}e\left[\frac{1}{2}e(11)\right] - \left[12.5 + \frac{2}{3}(d - 12.5)\right]\left[\frac{1}{2}(d - 12.5)(11)\right]}{d(11) - \frac{1}{2}e(11) - \frac{1}{2}(d - 12.5)(11)}$$

$$\overline{y} = \frac{\frac{1}{2}(11)[d(11)] - \frac{2}{3}(11)\left[\frac{1}{2}e(11)\right] - \left[\frac{1}{3}(11)\right]\left[\frac{1}{2}(d - 12.5\text{ m})(11\text{ m})\right]}{d(11) - \frac{1}{2}e(11) - \frac{1}{2}(d - 12.5)(11)}$$

Solving we find

$$\boxed{\overline{x} = 9.64\text{ m},\ \overline{y} = 4.60\text{ m}}$$

Problem 7.41 The area has elliptical boundaries. If $a = 30$ mm, $b = 15$ mm, and $\varepsilon = 6$ mm, what is the x coordinate of the centroid of the area?



Solution: The equation of the outer ellipse is

$$\frac{x^2}{(a+\varepsilon)^2} + \frac{y^2}{(b+\varepsilon)^2} = 1$$

and for the inner ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We will handle the problem by considering two solid ellipses

For any ellipse

$$\mathbf{x} = \frac{\int x\,dA}{\int dA} = \frac{\cancel{\frac{\beta}{\alpha}} \int_0^\alpha x\sqrt{\alpha^2 - x^2}\,dx}{\cancel{\frac{\beta}{\alpha}} \int \sqrt{\alpha^2 - x^2}\,dx}$$

From integral tables

$$\int x\sqrt{\alpha^2 - x^2}\,dx = -\frac{(\alpha^2 - x^2)^{3/2}}{3}$$

$$\int \sqrt{\alpha^2 - x^2}\,dx = \frac{x\sqrt{\alpha^2 - x^2}}{2} + \frac{\alpha^2}{2}\sin^{-1}\left(\frac{x}{\alpha}\right)$$

Substituting $\mathbf{x} = \dfrac{\left[-\left(\alpha^2 - x^2\right)^{3/2}\right]_0^\alpha}{\left[\dfrac{x\sqrt{\alpha^2 - x^2}}{3} + \dfrac{\alpha^2}{2}\sin\left(\dfrac{x}{\alpha}\right)\right]_0^\alpha}$

$$\mathbf{x} = \frac{[-0 + \alpha^3/3]}{\left[0 + \frac{\alpha^2}{2}\left(\frac{\pi}{2}\right) - 0 - 0\right]} = \frac{\alpha^3/3}{\alpha^2\pi/4}$$

$$\mathbf{x} = \frac{4\alpha}{3\pi}$$

Also Area $= \displaystyle\int dA = \frac{\beta}{\alpha}\int_0^\alpha \sqrt{\alpha^2 + x^2}\,dx$

$$= \frac{\beta}{\alpha}\left[\frac{x\sqrt{\alpha^2 - x^2}}{2} + \frac{\alpha^2}{2}\sin^{-1}\left(\frac{x}{\alpha}\right)\right]_0^\alpha$$

$$\text{Area} = \frac{\beta}{\alpha}\left(\frac{\alpha^2}{2}\right)\left(\frac{\pi}{2}\right) = \pi\alpha\beta/4$$

(The area of a full ellipse is $\pi\alpha\beta$ so this checks.

Now for the composite area.

For the outer ellipse, $\alpha = a + \varepsilon \quad \beta = b + \varepsilon$ and for the inner ellipse $\alpha = a \quad \beta = b$

Outer ellipse

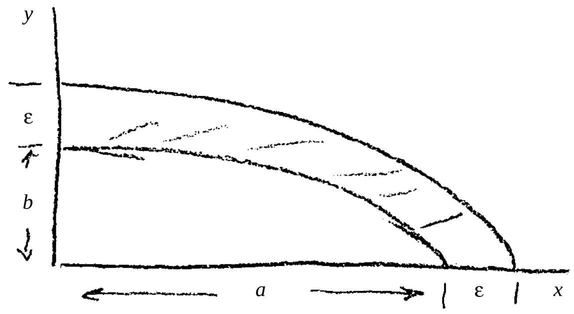
$$\mathbf{x}_1 = \frac{4(a+\varepsilon)}{3\pi}$$

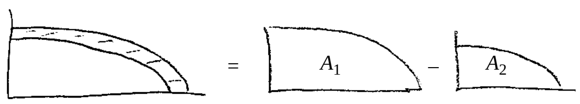
$$A_1 = \frac{\pi(a+\varepsilon)(b+\varepsilon)}{4}$$

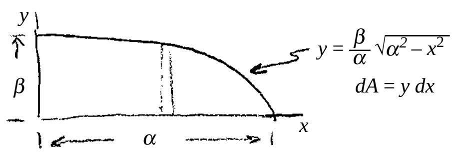
Inner Ellipse

$$\mathbf{x}_2 = \frac{4a}{3\pi}$$

$$A_2 = \frac{\pi ab}{4}$$







For the composite

$$\mathbf{x} = \frac{\mathbf{x}_1 A_1 - \mathbf{x}_2 A_2}{A_1 - A_2}$$

Substituting, we get

$\mathbf{x}_1 = 15.28$ mm $\qquad \mathbf{x}_2 = 12.73$ mm

$A_1 = 2375$ mm^2 $\qquad A_2 = 1414$ mm^2

and $\mathbf{x} = 19.0$ mm

Problem 7.42 By determining the x coordinate of the centroid of the area shown in Problem 7.41 in terms of a, b, and ε, and evaluating its limit as $\varepsilon \to 0$, show that the x coordinate of the centroid of a quarter-elliptical line is

$$\bar{x} = \frac{4a(a+2b)}{3\pi(a+b)}.$$

Solution: From the solution to 7.41, we have

$$\mathbf{x}_1 = \frac{4(a+\varepsilon)}{3\pi} \quad A_1 = \frac{\pi(a+\varepsilon)(b+\varepsilon)}{4}$$

$$\mathbf{x}_2 = \frac{4a}{3\pi} \qquad A_2 = \frac{\pi ab}{4}$$

$$\text{so } \mathbf{x}_1 A_1 = \frac{(a+\varepsilon)^2(b+\varepsilon)}{3}$$

$$x_2 A_2 = \frac{a^2 b}{3}$$

$$A_1 - A_2 = \frac{\pi}{4}(ab + a\varepsilon + b\varepsilon + \varepsilon^2 - ab)$$

$$A_1 - A_2 = \frac{\pi}{4}(a\varepsilon + b\varepsilon + \varepsilon^2)$$

$$(x_1 A_1 - x_2 A_2) = \tfrac{1}{3}(a^2 b + 2ab\varepsilon + b\varepsilon^2$$

$$+ a^2\varepsilon + 2a\varepsilon^2 + \varepsilon^3 - a^2 b)$$

$$(x_1 A_1 - x_2 A_2) = \tfrac{1}{3}((2ab + a^2)\varepsilon$$

$$+ (2a+b)\varepsilon^2 + \varepsilon^3)$$

Finally $\bar{x} = \dfrac{x_1 A_1 - x_2 A_2}{A_1 - A_2}$

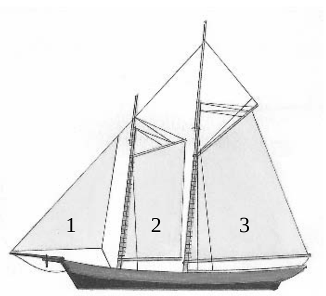
$$\bar{x} = \frac{\frac{1}{3}\left[(2ab + a^2) + (2a+b)\varepsilon + \varepsilon^2\right]\varepsilon}{\frac{\pi}{4}\left[(a+b)+\varepsilon\right]\varepsilon}$$

$$\bar{x} = \frac{4a(a+2b)}{3\pi(a+b)} + \frac{4(2\mathbf{a}+b)\varepsilon}{3\pi} + \frac{4}{3\pi}\varepsilon^2$$

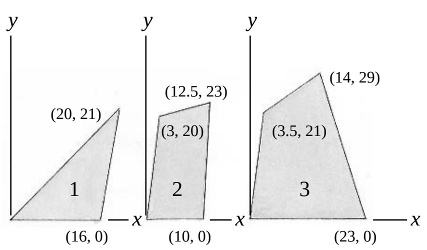
Taking the limit as $\varepsilon \to 0$

$$\bar{x} = \frac{4a(a+2b)}{3\pi(a+b)}$$

Problem 7.43 Three sails of a New York pilot schooner are shown. The coordinates of the points are in milimetres. Determine the centroid of sail 1.



(a)



(b)

Solution: Divide the object into three areas: (1) The triangle with altitude 21 mm and base 20 mm. (2) The triangle with altitude 21 mm and base $(20 - 16) = 4$mm, and (3) the composite sail. The areas and coordinates are:

(1) $A_1 = 210\ \text{mm}^2$,

$$\mathbf{x}_1 = \left(\frac{2}{3}\right) 20 = 13.33\ \text{mm},$$

$$\mathbf{y}_1 = \left(\frac{1}{3}\right) 21 = 7\ \text{mm}.$$

(2) $A_2 = 42\ \text{mm}^2$,

$$\mathbf{x}_2 = 16 + \left(\frac{2}{3}\right) 4 = 18.67\ \text{mm},$$

$$\mathbf{y}_2 = 7\ \text{mm}.$$

(3) The composite area: $A = A_1 - A_2 = 168\ \text{mm}^2$.
The composite centroid:

$$\boxed{\mathbf{x} = \frac{A_1\mathbf{x}_1 - A_2\mathbf{x}_2}{A} = 12\ \text{mm}},$$

$$\boxed{\mathbf{y} = \frac{A_1\mathbf{y}_1 - A_2\mathbf{y}_2}{A} = 7\ \text{mm}}$$

Problem 7.44 Determine the centroid of sail 2 in Problem 7.43.

Solution: Divide the object into five areas: (1) a triangle on the left with altitude 20 mm and base 3 mm, (2) a rectangle in the middle 23 mm by 9.5 mm, (3) a triangle at the top with base of 9.5 mm and altitude of 3 mm. (4) a triangle on the right with altitude of 23 mm and base of 2.5 mm. (5) the composite sail. The areas and centroids are:

(1) $A_1 = \frac{3(20)}{2} = 30 \text{ mm}^2,$

$\mathbf{x}_1 = \left(\frac{2}{3}\right) 3 = 2 \text{ mm},$

$\mathbf{y}_1 = \left(\frac{1}{3}\right) 20 = 6.67 \text{ mm}.$

(2) $A_2 = (23)(9.5) = 218.5 \text{ mm}^2,$

$\mathbf{x}_2 = 3 + \left(\frac{9.5}{2}\right) = 7.75 \text{ mm},$

$\mathbf{y}_2 = \frac{23}{2} = 11.5 \text{ mm}$

(3) $A_3 = \left(\frac{1}{2}\right)(3)(9.5) = 14.25 \text{ mm}^2,$

$\mathbf{x}_3 = 3 + \left(\frac{1}{3}\right) 9.5 = 6.167 \text{ mm},$

$\mathbf{y}_3 = 20 + \left(\frac{2}{3}\right) 3 = 22 \text{ mm}$

(4) $A_4 = \left(\frac{1}{2}\right)(2.5)(23) = 28.75 \text{ mm}^2,$

$\mathbf{x}_4 = 10 + \left(\frac{2}{3}\right)(2.5) = 11.67 \text{ mm},$

$\mathbf{y}_4 = \left(\frac{1}{3}\right) 23 = 7.66 \text{ mm}$

(5) The composite area: $A = A_1 + A_2 - A_3 - A_4 = 205.5 \text{ mm}^2.$ The composite centroid:

$$\boxed{\mathbf{x} = \frac{A_1\mathbf{x}_1 + A_2\mathbf{x}_2 - A_3\mathbf{x}_3 - A_4\mathbf{x}_4}{A} = 6.472 \text{ mm}},$$

$$\boxed{\mathbf{y} = \frac{A_1\mathbf{y}_1 + A_2\mathbf{y}_2 - A_3\mathbf{y}_3 - A_4\mathbf{y}_4}{A} = 10.603 \text{ mm}}$$

Problem 7.45 Determine the centroid of sail 3 in Problem 7.43.

Solution: Divide the object into six areas: (1) The triangle Oef, with base 3.5 mm and altitude 21 mm. (2) The rectangle Oabc, 14 mm by 29 mm. (3) The triangle beg, with base 10.5 mm and altitude 8 mm. (4) The triangle bcd, with base 9 mm and altitude 29 mm. (5) The rectangle agef 3.5 mm by 8 mm. (6) The composite, Oebd. The areas and centroids are:

(1) $A_1 = 36.75 \text{ mm}^2,$

$\mathbf{x}_1 = 1.167 \text{ mm},$

$\mathbf{y}_1 = 14 \text{ mm}.$

(2) $A_2 = 406 \text{ mm}^2,$

$\mathbf{x}_2 = 7 \text{ mm},$

$\mathbf{y}_2 = 14.5 \text{ mm}.$

(3) $A_3 = 42 \text{ mm}^2,$

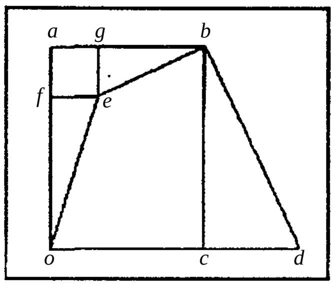
$\mathbf{x}_3 = 7 \text{ mm},$

$\mathbf{y}_3 = 26.33 \text{ mm}$

(4) $A_4 = 130.5 \text{ mm}^2,$

$\mathbf{x}_4 = 17 \text{ mm},$

$\mathbf{y}_4 = 9.67 \text{ mm}.$



(5) $A_5 = 28 \text{ mm}^2,$

$\mathbf{x}_5 = 1.75 \text{ mm},$

$\mathbf{y}_5 = 25 \text{ mm}.$

(6) The composite area:

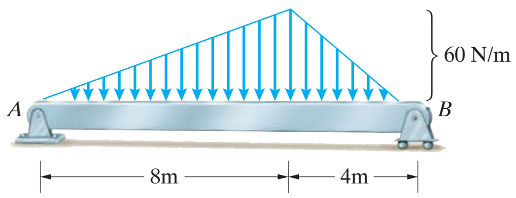
$A = -A_1 + A_2 - A_3 + A_4 - A_5 = 429.75 \text{ mm}^2.$

The composite centroid:

$$\boxed{\mathbf{x} = \frac{-A_1\mathbf{x}_1 + A_2\mathbf{x}_2 - A_3\mathbf{x}_3 + A_4\mathbf{x}_4 - A_5\mathbf{x}_5}{A} = 10.877 \text{ mm}}$$

$$\boxed{\mathbf{y} = \frac{-A_1\mathbf{y}_1 + A_2\mathbf{y}_2 - A_3\mathbf{y}_3 + A_4\mathbf{y}_4 - A_5 y_5}{A} = 11.23 \text{ mm}}$$

Problem 7.46 In Active Example 7.5, suppose that the distributed load is modified as shown. Determine the reactions on the beam at A and B.



Solution: We can treat the distributed load as two triangular distributed loads. Using the area analogy, the magnitude of the left one is $\frac{1}{2}$ (8 m)(60 N/m) = 240 N, and the magnitude of the right one is $\frac{1}{2}$ (4 m)(60 N/m) = 120 N. They must be placed at the centroids of the triangular distributions.

The equilibrium equations are

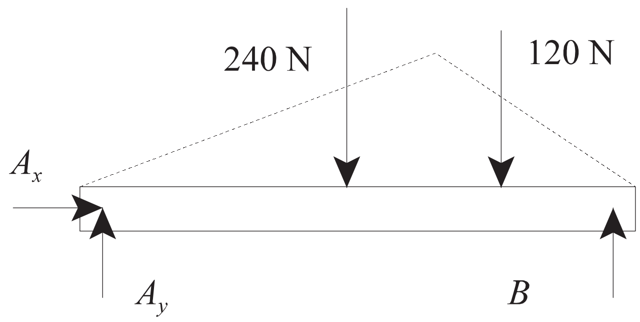
$$\Sigma F_x : A_x = 0,$$

$$\Sigma F_y : A_y + B - 240\text{ N} - 120\text{ N} = 0,$$

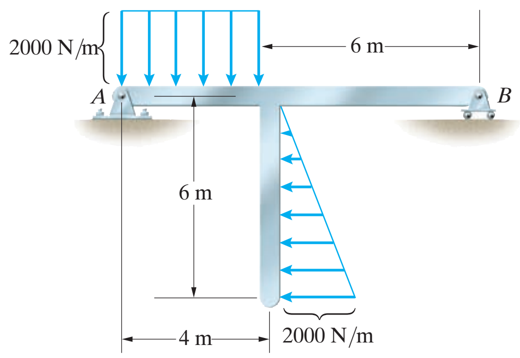
$$\Sigma M_A : (12\text{ m})\,B - [\tfrac{2}{3}\,(8\text{ m})](240\text{ N}) - [12\text{ m} - \tfrac{2}{3}\,(4\text{ m})](120\text{ N}) = 0.$$

Solving yields

$$\boxed{A_x = 0, \quad A_y = 160\text{ N}, \quad B = 200\text{ N}.}$$



Problem 7.47 Determine the reactions at A and B.



Solution: From the free-body diagram of the bar (with the distributed loads represented by equivalent force), the equilibrium equations are

$$\Sigma F_x : A_x - 6000\text{ N} = 0$$

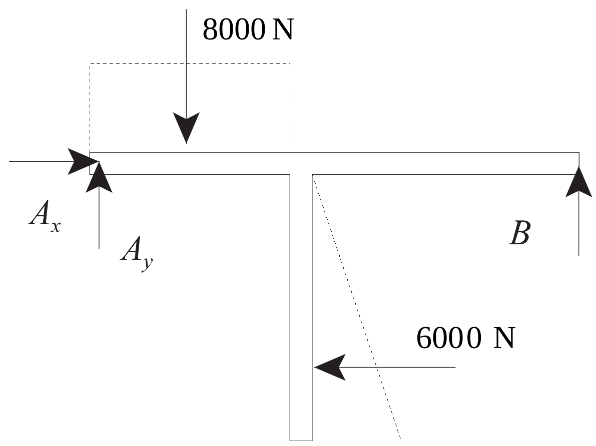
$$\Sigma F_y : A_y + B - 8000\text{ N} = 0$$

$$\Sigma M_A : -(8000\text{ N})\,(2\text{ m})$$

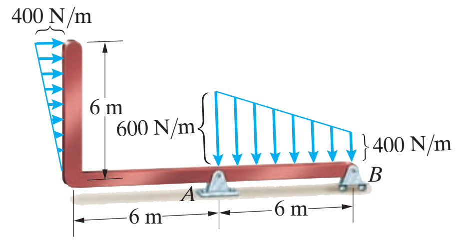
$$-\,(6000\text{ N})\,(4\text{ m}) + B(10\text{ m}) = 0$$

Solving yields

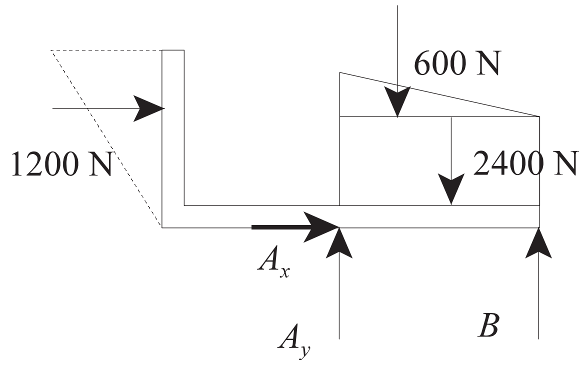
$$\boxed{A_x = 6000\text{ N}, \quad A_y = 4000\text{ N}, \quad B = 4000\text{ N}.}$$



Problem 7.48 In Example 7.6, suppose that the distributed loads are modified as shown. Determine the reactions on the beam at A and B.



Solution: The distributed loads are represented by three equivalent forces. The equilibrium equations are



$\Sigma F_x : 1200 \text{ N} + A_x = 0$

$\Sigma F_y : A_y + B - 2400 \text{ N} - 600 \text{ N} = 0$

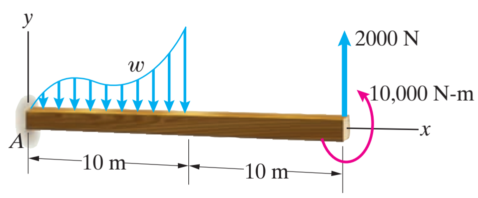
$\Sigma M_A : -(1200 \text{ N})\ (4 \text{ m}) = (600 \text{ N})\ (2 \text{ m})$

$\quad - (2400 \text{ N})\ (3 \text{ m}) + B\,(6 \text{ m}) = 0$

Solving yields

$$\boxed{A_x = -1200 \text{ N}, \quad A_y = 800 \text{ N}, \quad B = 2200 \text{ N}}$$

Problem 7.49 In Example 7.7, suppose that the distributed load acting on the beam from $x = 0$ to $x = 10$ m is given by $w = 350 + 0.3x^3$ N/m. (a) Determine the downward force and the clockwise moment about A exerted by the distributed load. (b) Determine the reactions at the fixed support.



Solution:

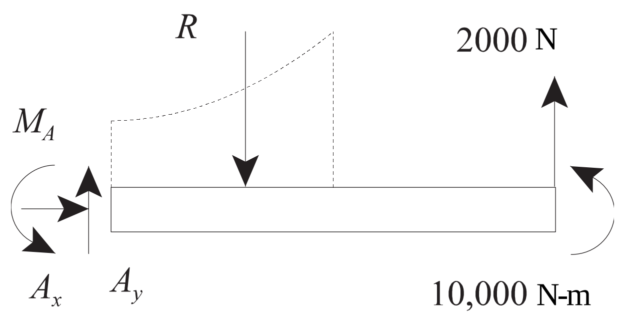
(a) The force and moment are

$$R = \int_0^{10} (350 + 0.3x^3)\, dx = 4250 \text{ N}$$

$$M = \int_0^{10} x(350 + 0.3x^3)\, dx = 23500 \text{ N-m}$$

$$\boxed{R = 4250 \text{ N}, M = 23{,}500 \text{ N-m}}$$

(b) The equilibrium equations are



$\Sigma F_x : A_x = 0,$

$\Sigma F_y : A_y - 4250 \text{ N} + 2000 \text{ N} = 0,$

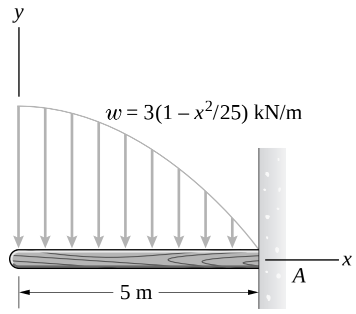
$\Sigma M_A : M_A - 23500 \text{ N-m}$

$\quad + (2000 \text{ N})(20 \text{ m})$

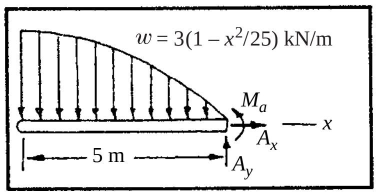
$\quad + 10{,}000 \text{ N-m} = 0.$

Solving yields $\boxed{A_x = 0, \ A_y = 2250 \text{ N}, \ M_A = -26{,}500 \text{ N-m}}$

Problem 7.50 Determine the reactions at the fixed support A.



Solution: The free-body diagram of the beam is: The downward force exerted by the distributed load is



$$\int_L w\,dx = \int_0^5 3\left(1 - \frac{x^2}{25}\right)\,dx$$

$$= 3\left[x - \frac{x^3}{75}\right]_0^5 = 10 \text{ kN}.$$

The clockwise moment about the left end of the beam due to the distributed load is

$$\int_L xw\,dx = \int_0^5 3\left(x - \frac{x^3}{25}\right)\,dx$$

$$= 3\left[\frac{x^2}{2} - \frac{x^4}{100}\right]_0^5 = 18.75 \text{ kN-m}.$$

From the equilibrium equations

$$\sum F_x = A_x = 0,$$

$$\sum F_y = A_y - 10 = 0,$$

$$\sum m_{(\text{leftend})} = M_a + 5A_y - 18.75 = 0,$$

we obtain

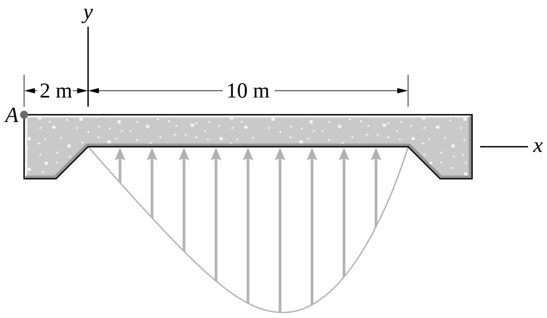
$$A_x = 0,$$

$$A_y = 10 \text{ kN},$$

and $M_a = -31.25$ kN-m.

Problem 7.51 An engineer measures the forces exerted by the soil on a 10-m section of a building foundation and finds that they are described by the distributed load $w = -10x - x^2 + 0.2x^3$ kN/m.

(a) Determine the magnitude of the total force exerted on the foundation by the distributed load.
(b) Determine the magnitude of the moment about A due to the distributed load.



Solution:

(a) The total force is

$$F = -\int_0^{12} (10x + x^2 - 0.2x^3)\,dx$$

$$= \left[-5x^2 - \frac{x^3}{3} + \frac{0.2}{4}x^4\right]_0^{10}$$

$|F| = 333.3$ kN

(b) The moment about the origin is

$$M = -\int_0^{10} (10x + x^2 - 0.2x^3)x\,dx$$

$$= \left[-\frac{10}{3}x^3 - \frac{1}{4}x^4 + \frac{0.2}{5}x^5\right]_0^{10},$$

$|M| = 1833.33$ kN.

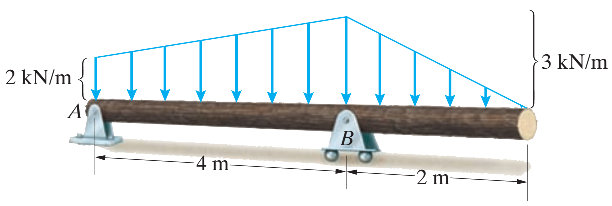
The distance from the origin to the equivalent force is

$$d = \frac{|M|}{F} = 5.5 \text{ m},$$

from which

$|M_A| = (d + 2)F = 2500$ kN m.

Problem 7.52 Determine the reactions on the beam at A and B.



Solution: Replace the distributed load with three equivalent single forces.

The equilibrium equations

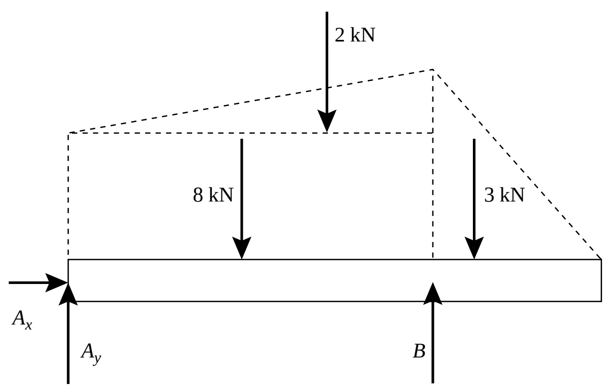
$$\sum F_x : A_x = 0$$

$$\sum F_y : A_y + B - 8 \text{ kN} - 2 \text{ kN} - 3 \text{ kN} = 0$$

$$\sum M_A : B(4 \text{ m}) - (8 \text{ kN})(2 \text{ m}) - (2 \text{ kN})\left(\tfrac{2}{3}4 \text{ m}\right)$$

$$- (3 \text{ kN})\left(4 \text{ m} + \tfrac{1}{3}2 \text{ m}\right) = 0$$

$$\boxed{A_x = 0,\ A_y = 4.17 \text{ kN}\ B = 8.83 \text{ kN}}$$

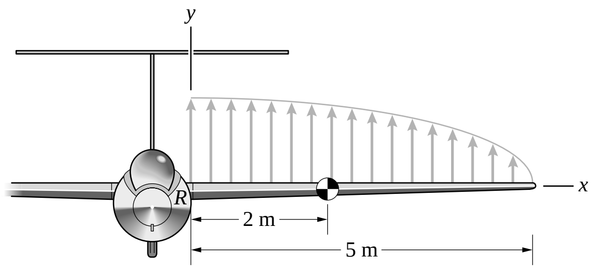


Problem 7.53 The aerodynamic lift of the wing is described by the distributed load

$$w = -300\sqrt{1 - 0.04x^2} \text{ N/m.}$$

The mass of the wing is 27 kg, and its center of mass is located 2 m from the wing root R.

(a) Determine the magnitudes of the force and the moment about R exerted by the lift of the wing.
(b) Determine the reactions on the wing at R.



Solution:

(a) The force due to the lift is

$$F = -w = \int_0^5 300(1 - 0.04x^2)^{1/2}\,dx,$$

$$F = \frac{300}{5}\int_0^5 (25 - x^2)^{1/2}\,dx$$

$$F = 60\left[\frac{x\sqrt{25 - x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5 = 375\pi \text{ N},$$

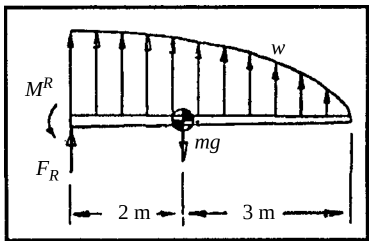
$|F| = 1178.1$ N.

The moment about the root due to the lift is

$$M = 300\int_0^5 (1 - 0.04x^2)^{1/2} x\,dx,$$

$$M = -60\left[\frac{(25 - x^2)^{3/2}}{3}\right]_0^5 = \frac{60(25)^{3/2}}{3} = 2500$$

$|M| = 2500$ Nm.



(b) The sum of the moments about the root:

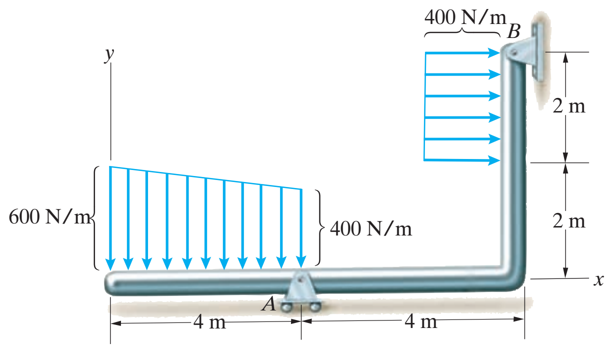
$$\sum M = M^R + 2500 - 27g(2) = 0,$$

from which $M^R = -1970$ N-m. The sum of forces

$$\sum F_y = F_R + 1178.1 - 27g = 0,$$

from which $F_R = -1178.1 + 27g = -913.2$ N

Problem 7.54 Determine the reactions on the bar at A and B.



Solution: First replace the distributed loads with three equivalent forces.

The equilibrium equations

$$\sum F_x : B_x + 800\ \text{N} = 0$$

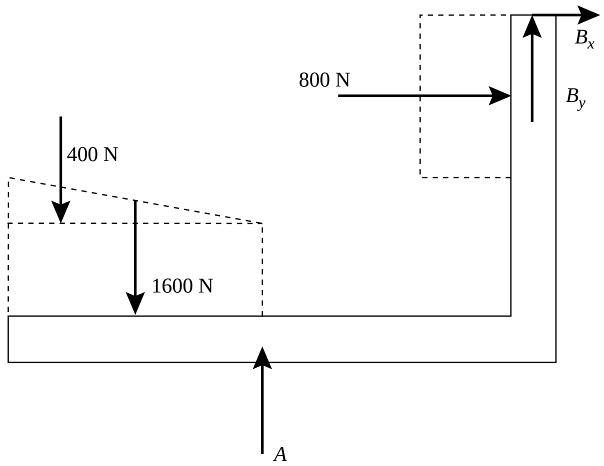
$$\sum M_B : (800\ \text{N})(1\ \text{m}) - A(4\ \text{m}) + (1600\ \text{N})(6\ \text{m})$$

$$+\ (400\ \text{N})(6.67\ \text{m}) = 0$$

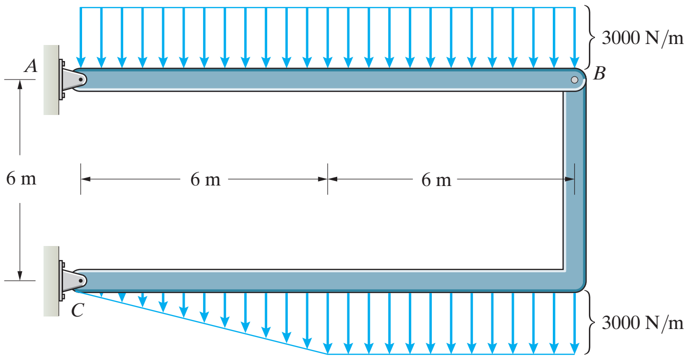
$$\sum F_y : A + B_y - 1600\ \text{N} - 400\ \text{N} = 0$$

Solving:

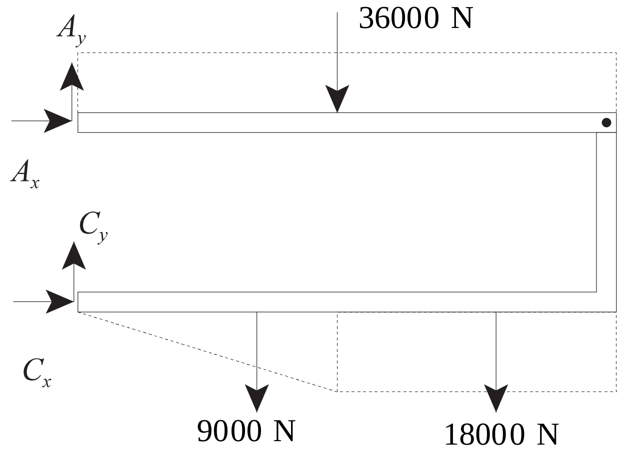
$$\boxed{A = 3267\ \text{N},\ B_x = -800\ \text{N},\ B_y = -1267\ \text{N}}$$



Problem 7.55 Determine the reactions on member AB at A and B.



Solution: From the free-body diagram of the entire structure (with the distributed loads represented by equivalent forces), one of the equilibrium equations is



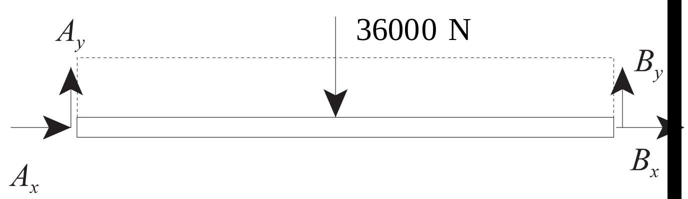
$$\Sigma M_C : -A_x\ (6\text{ m})$$

$$-\ (36000\text{ N})\ (6\text{ m})$$

$$-\ (9000\text{ N})\ (4\text{ m})$$

$$-\ (18000\text{ N})\ (9\text{ m}) = 0$$

From the free-body diagram of member AB we have the equilibrium equations



$$\Sigma F_x : A_x + B_x = 0$$

$$\Sigma F_y : A_y + B_y - 36000\text{ N} = 0$$

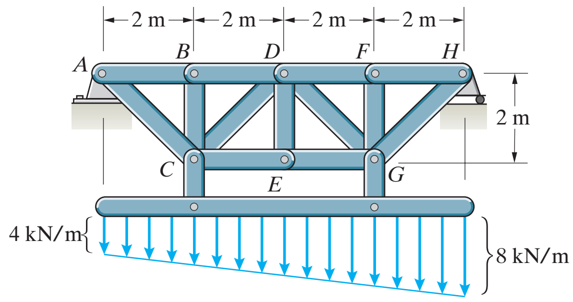
$$\Sigma M_A : B_y\ (12\text{ m})$$

$$-\ (36000\text{ N})\ (6\text{ m}) = 0$$

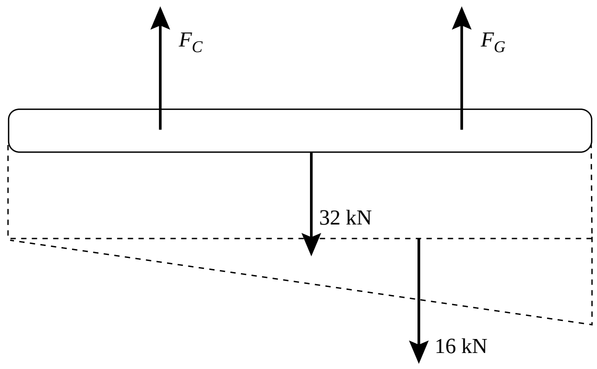
Solving yields

$$\boxed{\begin{array}{l} A_x = -69000\text{ N},\ \ A_y = 18000\text{ N}, \\ B_x = 69000\text{ N},\ \ B_y = 18000\text{ N}. \end{array}}$$

Problem 7.56 Determine the axial forces in members *BD*, *CD*, and *CE* of the truss and indicate whether they are in tension (T) or compression (C).



Solution: We start by analyzing the horizontal bar with the distributed load



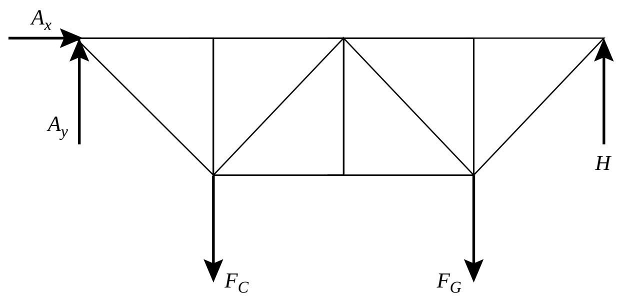
$$\sum M_G : (16\text{ kN})(0.667\text{ m}) + (32\text{ kN})(2\text{ m})$$

$$-F_C(4\text{ m}) = 0 \Rightarrow F_C = 18.67\text{ kN}$$

$$\sum F_y = F_C + F_G - 32\text{ kN} - 16\text{ kN} = 0$$

$$\Rightarrow F_G = 29.33\text{ kN}$$

Now work with the whole structure in order to find the reactions at *A*

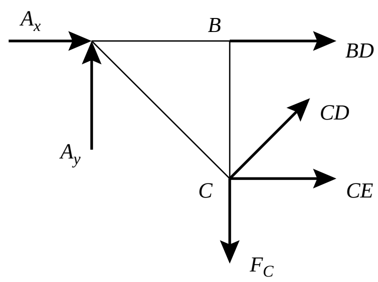


$$\sum F_x : A_x = 0 \Rightarrow A_x = 0$$

$$\sum M_H : F_G\ (2\text{ m}) + F_C\ (6\text{ m})$$

$$-A_y\ (8\text{ m}) = 0 \Rightarrow A_y = 21.3\text{ kN}$$

Finally, cut through the truss and look at the left section



$$\sum M_C : -A_x\ (2\text{ m}) - A_y\ (2\text{ m}) - BD(2\text{ m}) = 0$$

$$\sum M_D : -A_y\ (4\text{ m}) + F_C\ (2\text{ m}) + CE(2\text{ m}) = 0$$

$$\sum F_y : A_y - F_C + \frac{1}{\sqrt{2}}CD = 0$$

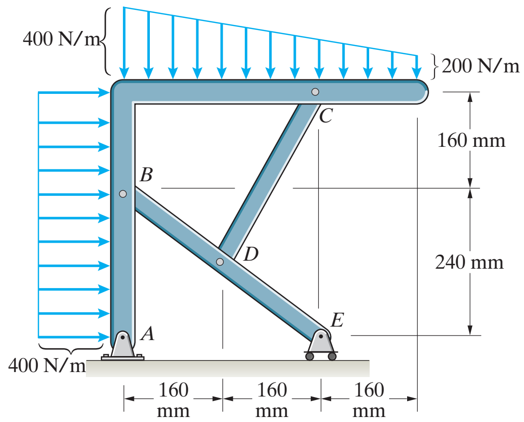
Solving we find

$$BD = -21.3\text{ kN},\ \ CD = -3.77\text{ kN},\ \ CE = 24\text{ kN}$$

In summary:

$$\boxed{BD = 21.3\text{ kN}(C),\ \ CD = 3.77\text{ kN}(C),\ \ CE = 24\text{ kN}(T)}$$

Problem 7.57 Determine the reactions on member *ABC* at *A* and *B*.



Solution: Work on the entire structure first to find the reactions at A. Replace the distributed forces with equivalent concentrated forces

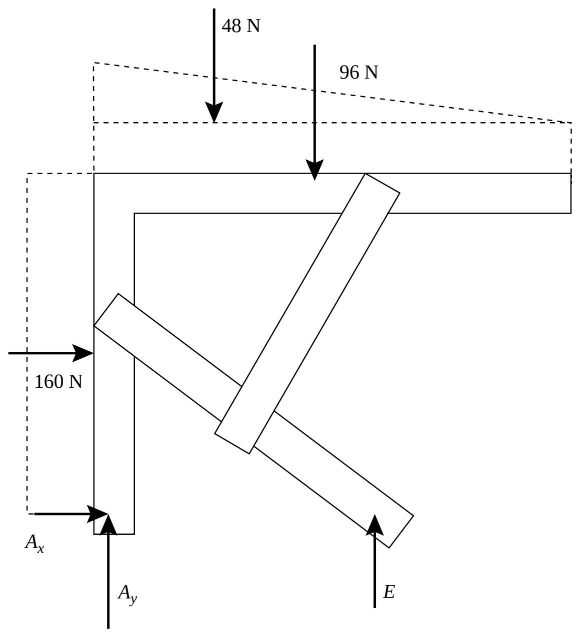
$$\sum F_x : A_x + 160 \text{ N} = 0$$

$$\sum M_E : (96 \text{ N})(0.08 \text{ m}) + (48 \text{ N})(0.16 \text{ m})$$

$$- (160 \text{ N})(0.2 \text{ m}) - A_y(0.32 \text{ m}) = 0$$

Solving:

$$A_x = -160 \text{ N}, \; A_y = -52 \text{ N}$$



Now look at body *ABC*. Take advantage of the two-force body *CD*.

$$\sum M_B : A_x(0.24 \text{ m}) + (160 \text{ N})(0.04 \text{ m})$$

$$- (48 \text{ N})(0.16 \text{ m}) - (96 \text{ N})(0.24 \text{ m})$$

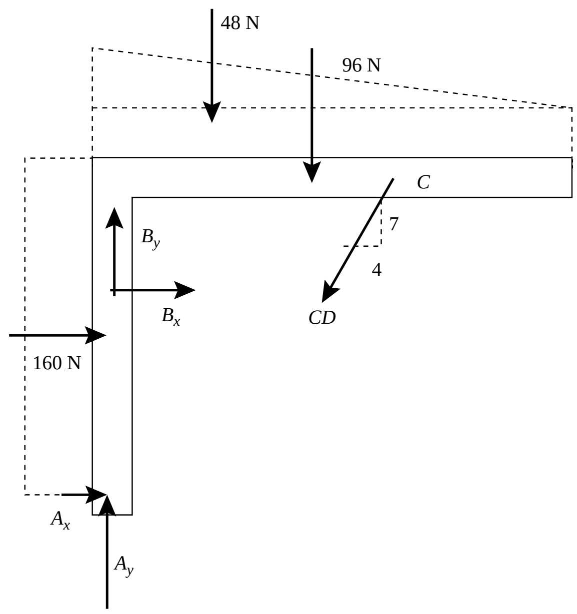
$$- \frac{7}{\sqrt{65}} CD(0.32 \text{ m}) + \frac{4}{\sqrt{65}} CD(0.16 \text{ m}) = 0$$

$$\sum F_x : A_x + B_x + 160 \text{ N} - \frac{4}{\sqrt{65}} CD = 0$$

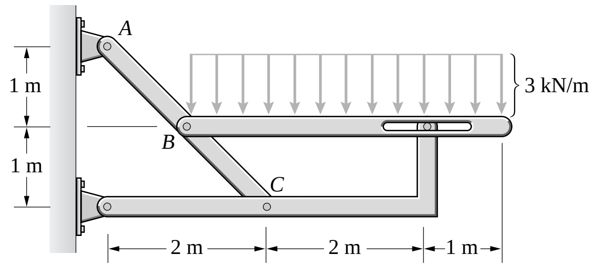
$$\sum F_y : A_y + B_y - 48 \text{ N} - 96 \text{ N} - \frac{7}{\sqrt{65}} CD = 0$$

Solving:

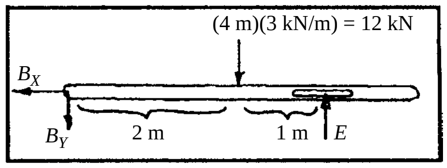
$$\boxed{\begin{array}{l} A_x = -160 \text{ N}, \; A_y = -52 \text{ N} \\ B_x = -157 \text{ N}, \; B_y = -78.4 \text{ N} \end{array}}$$



Problem 7.58 Determine the forces on member *ABC* of the frame.



Solution: The free body diagram of the member on which the distributed load acts is



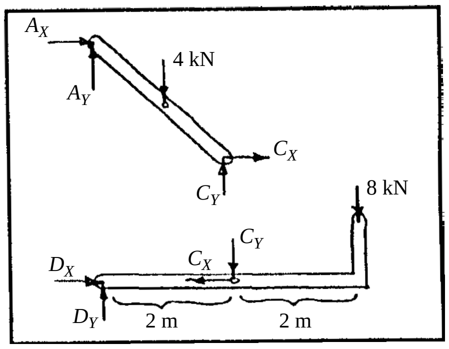
From the equilibrium equations

$$\sum F_x = B_x = 0,$$

$$\sum F_y = B_y + E - 12 = 0,$$

$$\sum m_{(\text{leftend})} = 3E - (2)(12) = 0,$$

we find that $B_x = 0$, $B_y = 4$ kN, and $E = 8$ kN. From the lower fbd, writing the equilibrium equation



$$\sum m_{(\text{leftend})} = -2C_y - (4)(8) = 0,$$

we obtain $C_y = -16$ kN. Then from the middle free body diagram, we write the equilibrium equations

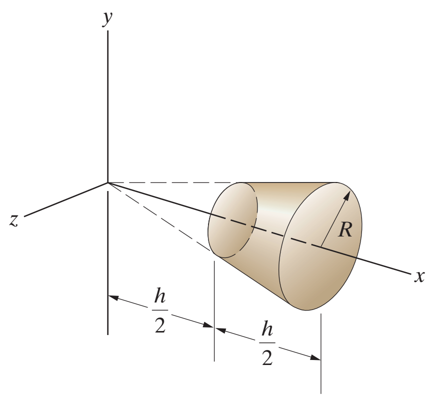
$$\sum F_x = A_x + C_x = 0,$$

$$\sum F_y = A_y - 4 - 16 = 0,$$

$$\sum m_{(\text{rightend})} = -2A_x - 2A_y + (1)(4) = 0$$

obtaining $A_x = -18$ kN, $A_y = 20$ kN, $C_x = 18$ kN.

Problem 7.59 Use the method described in Active Example 7.8 to determine the centroid of the truncated cone.



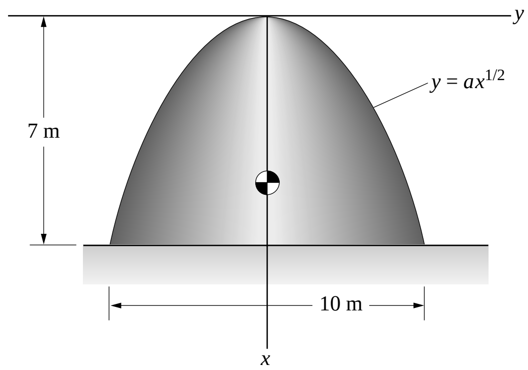
Solution: Just as in Active Example 7.8, the volume of the disk element is

$$dV = \pi\left(\frac{R}{h}x\right)^2 dx$$

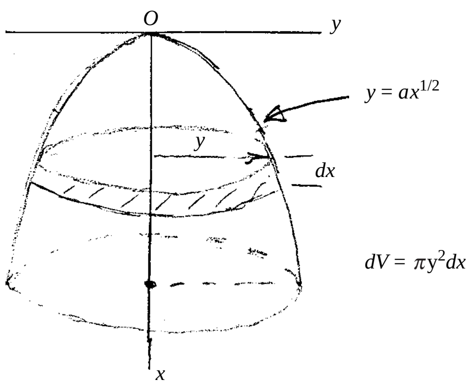
the x coordinate of the centroid is

$$\bar{x} = \frac{\int_V x dV}{\int_V dV} = \frac{\int_{h/2}^{h} x\pi\left(\frac{R}{h}x\right)^2 x}{\int_{h/2}^{h} \pi\left(\frac{R}{h}x\right)^2 x} = \frac{45}{56}h \qquad \boxed{\bar{x} = \frac{45}{56}h}$$

Problem 7.60 A grain storage tank has the form of a surface of revolution with the profile shown. The height of the tank is 7 m and its diameter at ground level is 10 m. Determine the volume of the tank and the height *above ground level* of the centroid of its volume.



Solution:



$$dV = \pi y^2\,dx$$

$$\mathbf{x} = \frac{\int_0^7 \chi\pi y^2\,dx}{\int_0^7 \pi y^2\,dx} = \frac{\int_0^7 \chi\pi a^2 x\,dx}{\int_0^7 \pi a^2 x\,dx}$$

$$\mathbf{x} = \frac{[x^3/3]_0^7}{[x^2/2]_0^7} = 4.67\text{ m}$$

The height of the centroid above the ground is 7 m $-$ $\mathbf{x}$

$h = 2.33$ m

The volume is

$$V = \int_0^7 \pi a^2 x\,dx = \pi a^2\left(\frac{49}{2}\right)\ \text{m}^3$$

To determine a,

$y = 5,$ m when $x = 7$ m.

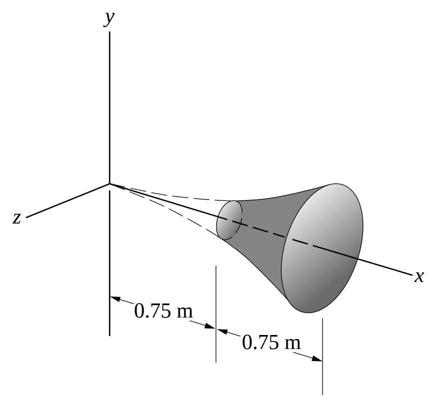
$y = ax^{1/2},\ 5 = a\sqrt{7}$

$a = 5/\sqrt{7}a^2 = 25/7$

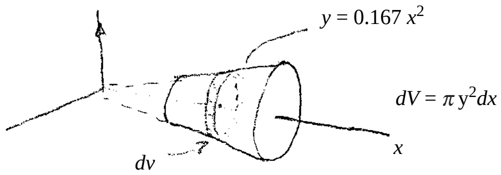
$$V = \pi\left(\frac{25}{7}\right)\left(\frac{49}{2}\right) = 275\text{ m}^3$$

$V = 275\text{ m}^3$

Problem 7.61 The object shown, designed to serve as a pedestal for a speaker, has a profile obtained by revolving the curve $y = 0.167x^2$ about the x axis. What is the x coordinate of the centroid of the object?



Solution:



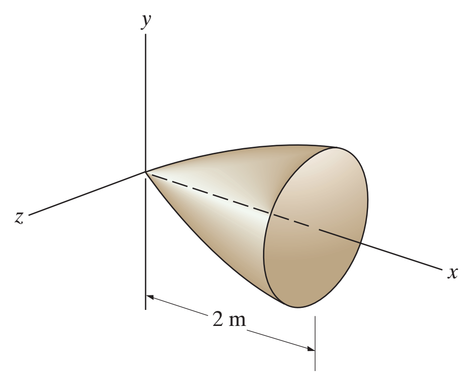
$$\mathbf{x} = \frac{\int_V x\,dV}{\int_V dV} = \frac{\int_{0.75}^{1.50} x\pi(0.167x^2)^2\,dx}{\int_{0.75}^{1.50} \pi(0.167x^2)^2\,dx}$$

$$\mathbf{x} = \frac{\pi(0.167)^2}{\pi(0.167)^2} \cdot \frac{\int_{0.75}^{1.5} x^5\,dx}{\int_{0.75}^{1.5} x^4\,dx} = \frac{\left[x^6/6\right]_{0.75}^{1.5}}{\left[x^5/5\right]_{0.75}^{1.5}}$$

$\mathbf{x} = 1.27$ m

Problem 7.62 The volume of a nose cone is generated by rotating the function $y = x - 0.2x^2$ about the x axis.

(a) What is the volume of the nose cone?
(b) What is the x coordinate of the centroid of the volume?

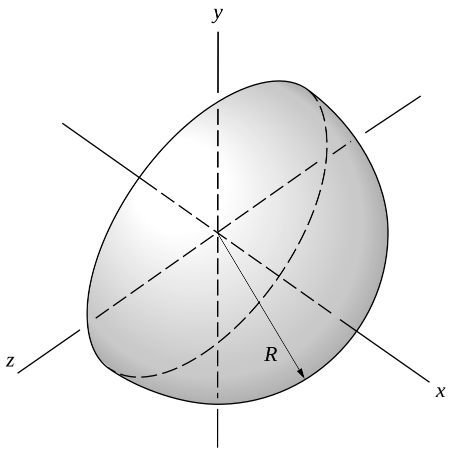


Solution:

(a) $$\boxed{V = \int_0^{2\text{ m}} \pi y^2 dx = \int_0^{2\text{ m}} \pi(x - 0.2x^2)^2 dx = 4.16\text{ m}^3}$$

(b) $$\boxed{\bar{x} = \frac{\int_0^{2\text{ m}} x(\pi y^2 dx)}{V} = \frac{\int_0^{2} \pi x(x - 0.2x^2)^2 dx}{4.155\text{ m}^3} = 1.411\text{ m}}$$

Problem 7.63 Determine the centroid of the hemispherical volume.



Solution: The equation of the surface of a sphere is $x^2 + y^2 + z^2 = R^2$.

The volume: The element of volume is a disk of radius ρ and thickness dx. The radius of the disk at any point within the hemisphere is $\rho^2 = y^2 + z^2$. From the equation of the surface of the sphere, $\rho^2 = (R^2 - x^2)$. The area is $\pi\rho^2$, and the element of volume is $dV = \pi(R^2 - x^2)\,dx$, from which

$$V = \frac{V_{\text{sphere}}}{2} = \frac{2\pi}{3}R^3.$$

The x-coordinate is:

$$\int_V x\,dV = \pi\int_0^R (R^2 - x^2)x\,dx$$

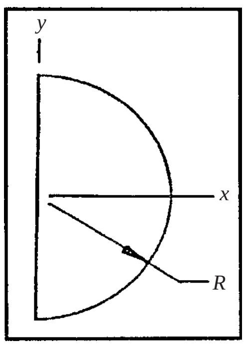
$$= \pi\left[\frac{R^2x^2}{2} - \frac{x^4}{4}\right]_0^R$$

$$= \frac{\pi}{4}R^4.$$

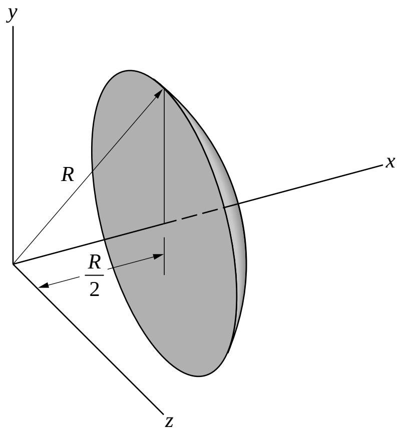
Divide by the volume:

$$\mathbf{x} = \left(\frac{\pi R^4}{4}\right)\left(\frac{3}{2\pi R^3}\right) = \frac{3}{8}R.$$

By symmetry, the y- and z-coordinates of the centroid are zero.



Problem 7.64 The volume consists of a segment of a sphere of radius R. Determine its centroid.



Solution: *The volume:* The element of volume is a disk of radius ρ and thickness dx. The area of the disk is $\pi\rho^2$, and the element of volume is $\pi\rho^2\,dx$. From the equation of the surface of a sphere (see solution to Problem 7.63) $\rho^2 = R^2 - x^2$, from which the element of volume is $dV = \pi(R^2 - x^2)\,dx$. Thus

$$V = \int_V dV = \pi \int_{R/2}^{R} (R^2 - x^2)\,dx$$

$$= \pi \left[R^2 x - \frac{x^3}{3} \right]_{R/2}^{R} = \left(\frac{5\pi}{24} \right) R^3.$$

The x-coordinate:

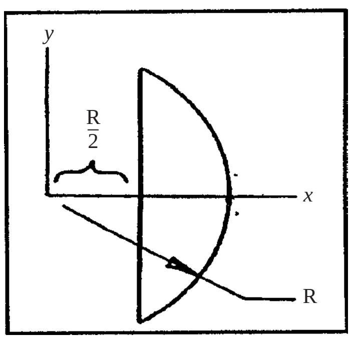
$$\int_V x\,dV = \pi \int_{R/2}^{R} (R^2 - x^2) x\,dx$$

$$= \pi \left[\frac{R^2 x^2}{2} - \frac{x^4}{4} \right]_{R/2}^{R} = \frac{9\pi}{64} R^4.$$

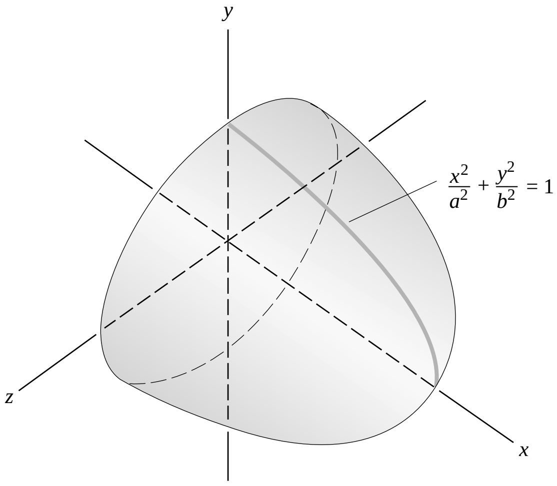
Divide by the volume:

$$\mathbf{x} = \left(\frac{9\pi R^4}{64} \right) \left(\frac{24}{5\pi R^3} \right) = \frac{27}{40} R = 0.675R.$$

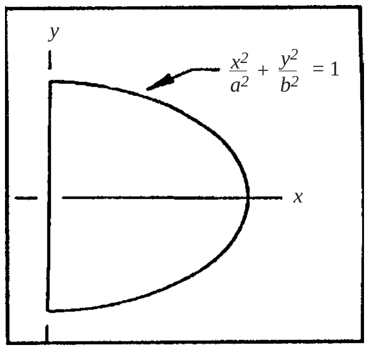
By symmetry the y- and z-coordinates are zero.



Problem 7.65 A volume of revolution is obtained by revolving the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x axis. Determine its centroid.



Solution: *The volume:* The element of volume is a disk of radius y and thickness dx. The area of the disk is πy^2. From the equation for the surface of the ellipse,



$$\pi y^2 = \pi b^2 \left(1 - \frac{x^2}{a^2}\right)$$

and $dV = \pi y^2\, dx = \pi b^2 \left(1 - \frac{x^2}{a^2}\right) dx$,

from which

$$V = \int_V dV = \pi b^2 \int_0^a \left(1 - \frac{x^2}{a^2}\right) dx$$

$$= \pi b^2 \left[x - \frac{x^3}{3a^2}\right]_0^a = \frac{2\pi b^2 a}{3}.$$

The x-coordinate:

$$\int_V x\, dV = \pi b^2 \int_0^a \left(1 - \frac{x^2}{a^2}\right) x\, dx$$

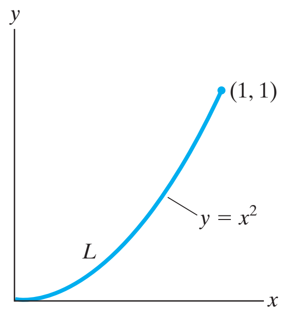
$$= \pi b^2 \left[\frac{x^2}{2} - \frac{x^4}{4a^2}\right]_0^a = \frac{\pi b^2 a^2}{4}.$$

Divide by volume:

$$\mathbf{x} = \left(\frac{\pi b^2 a^2}{4}\right)\left(\frac{3}{2\pi b^2 a}\right) = \left(\frac{3}{8}\right) a.$$

By symmetry, the y- and z-coordinates of the centroid are zero.

Problem 7.66 In Example 7.9, determine the y coordinate of the centroid of the line.

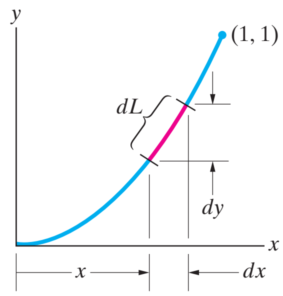


Solution: The expression derived in Example 7.9 for the element dL of the line in terms of x is

$$dL = \sqrt{1+4x^2}\,dx$$

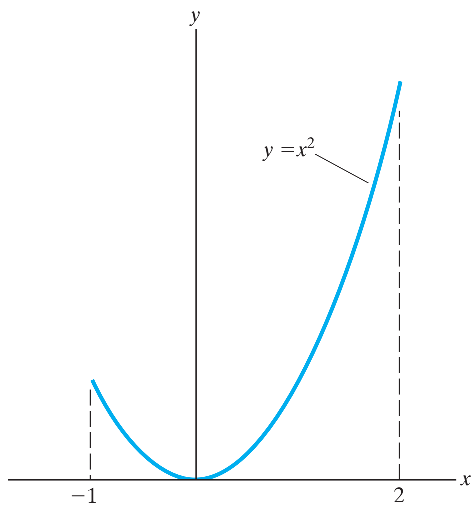
The y coordinate of the centroid is

$$\overline{y} = \frac{\int_L y\,dL}{\int_L dL} = \frac{\int_0^1 x^2\sqrt{1+4x^2}\,dx}{\int_0^1 \sqrt{1+4x^2}\,dx} = 0.410$$



$\boxed{\overline{y} = 0.410}$

Problem 7.67 Determine the coordinates of the centroid of the line.



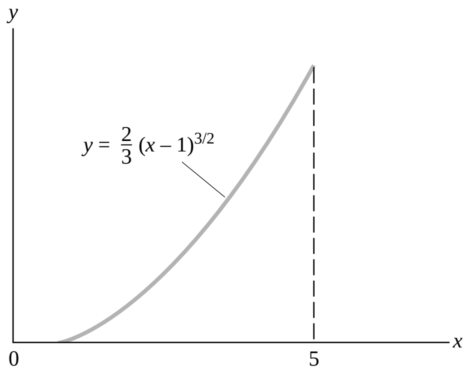
Solution:

$$\overline{x} = \frac{\int_{-1}^{2} x\,ds}{\int_{-1}^{2} ds} = \frac{\int_{-1}^{2} x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{\int_{-1}^{2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx} = \frac{\int_{-1}^{2} x\sqrt{1+(2x)^2}\,dx}{\int_{-1}^{2}\sqrt{1+(2x)^2}\,dx}$$

$$\overline{y} = \frac{\int_{-1}^{2} y\,ds}{\int_{-1}^{2} ds} = \frac{\int_{-1}^{2} y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{\int_{-1}^{2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx} = \frac{\int_{-1}^{2} x^2\sqrt{1+(2x)^2}\,dx}{\int_{-1}^{2}\sqrt{1+(2x)^2}\,dx}$$

$$\boxed{\begin{array}{l}\overline{x} = 0.801\\ \overline{y} = 1.482\end{array}}$$

Problem 7.68 Determine the x coordinate of the centroid of the line.



Solution: *The length:* Noting that $\frac{dy}{dx} = (x-1)^{1/2}$, the element of length is

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{x}\, dx$$

from which

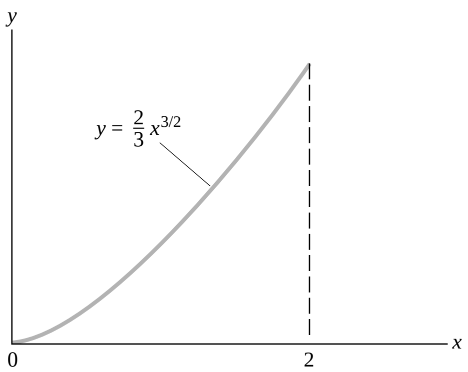
$$L = \int_L dL = \int_1^5 (x)^{1/2}\, dx = \left[\frac{2}{3}(x)^{3/2}\right]_1^5 = 6.7869.$$

The x-coordinate:

$$\int_L x\, dL = \int_0^5 x^{3/2}\, dx = \left[\frac{2}{5}x^{5/2}\right]_1^5 = 21.961.$$

Divide by the length: $\mathbf{x} = \frac{21.961}{6.7869} = 3.2357$

Problem 7.69 Determine the x coordinate of the centroid of the line.



Solution: *The length:* Noting that $\frac{dy}{dx} = x^{1/2}$ the element of length is

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1+x}\, dx$$

from which

$$L = \int_L dL = \int_0^2 (1+x)^{1/2}\, dx = \left[\frac{2}{3}(1+x)^{3/2}\right]_0^2 = 2.7974$$

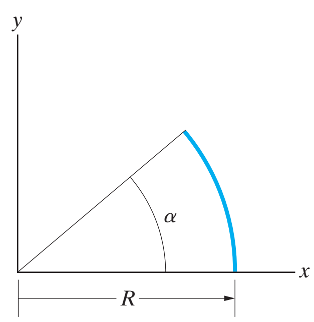
The x-coordinate:

$$\int_L x\, dL = \int_0^2 x(1+x)^{1/2}\, dx = 2\left[\frac{(1+x)^{5/2}}{5} - \frac{(1+x)^{3/2}}{3}\right]_0^2$$

$$= 2\left[\frac{3^{5/2}}{5} - \frac{3^{3/2}}{3} - \left(\frac{1}{5}\right) + \left(\frac{1}{3}\right)\right] = 3.0379.$$

Divide by the length: $\mathbf{x} = 1.086$

Problem 7.70 Use the method described in Example 7.10 to determine the centroid of the circular arc.



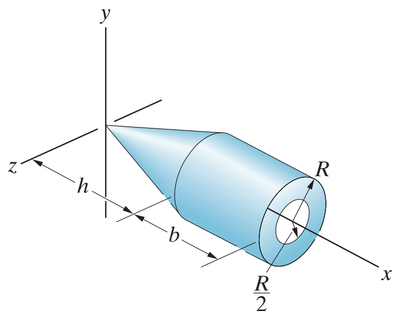
Solution: The length of the differential line element of the circular arc is $dL = Rd\theta$. The coordinates of the centroid are

$$\bar{x} = \frac{\int_L x dL}{\int_L dL} = \frac{\int_0^\alpha (R\cos\theta)\, Rd\theta}{\int_0^\alpha Rd\theta} = \frac{R\sin\alpha}{\alpha}$$

$$\bar{y} = \frac{\int_L y dL}{\int_L dL} = \frac{\int_0^\alpha (R\sin\theta)\, Rd\theta}{\int_0^\alpha Rd\theta} = \frac{R\,(1-\cos\alpha)}{\alpha}$$

Thus $\boxed{\bar{x} = \frac{R\sin\alpha}{\alpha}, \quad \bar{y} = \frac{R(1-\cos\alpha)}{\alpha}}$

Problem 7.71 In Active Example 7.11, suppose that the cylinder is hollow with inner radius $R/2$ as shown. If the dimensions $R = 6$ cm, $h = 12$ cm, and $b = 10$ cm, what is the x coordinate of the centroid of the volume?



Solution: Let the cone be volume 1, let the *solid* cylinder be volume 2, and let the cylindrical hole be volume 3. The volumes and the x coordinates of the their centroids are

$V_1 = \frac{1}{3}\pi R^2 h, \quad \bar{x}_1 = \frac{3}{4}h,$

$V_2 = \pi R^2 b, \quad \bar{x}_2 = h + \frac{1}{2}b,$

$V_3 = -\pi(\frac{1}{2}R)^2\, b, \ \bar{x}_3 = h + \frac{1}{2}b$

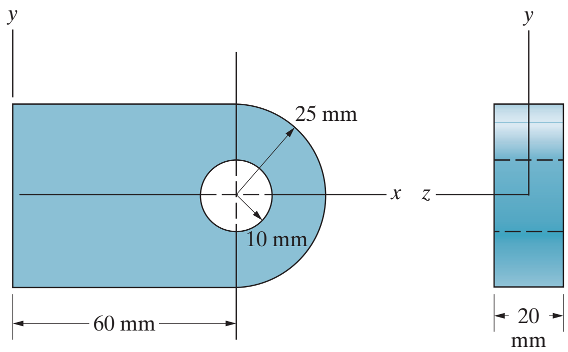
The x coordinate of the centroid of the composite volume is

$$\bar{x} = \frac{\bar{x}_1 V_1 + \bar{x}_2 V_2 + \bar{x}_3 V_3}{V_1 + V_2 + V_3}$$

$$= \frac{\left(\frac{3}{4}h\right)\left(\frac{1}{3}\pi R^2 h\right) + \left(h + \frac{1}{2}b\right)(\pi R^2 b) + \left(h + \frac{1}{2}b\right)\left(-\pi\left(\frac{1}{2}R\right)^2 b\right)}{\frac{1}{3}\pi R^2 h + \pi R^2 b - \pi\left(\frac{1}{2}R\right)^2 b}$$

Substituting the values for R, h, and b, we have $\boxed{\bar{x} = 14.2 \text{ cm.}}$

Problem 7.72 Use the procedure described in Example 7.12 to determine the x component of the centroid of the volume.



Solution: Let the rectangular part without the cutout be volume 1, let the semicylindrical part be volume 2, and let the cylindrical hole be volume 3. The volumes and the x coordinates of their centroids are

$V_1 = (60)\ (50)\ (20)\ \text{mm}^3, \quad \bar{x}_1 = 30 \text{ mm},$

$V_2 = \frac{1}{2}\pi(25)^2\ (20)\ \text{mm}^3, \quad \bar{x}_2 = 60 + \frac{4(25)}{3\pi} \text{ mm},$

$V_3 = -\pi(10)^2\ (20)\ \text{mm}^3, \quad \bar{x}_3 = 60 \text{ mm}.$

The x coordinate of the centroid of the composite volume is

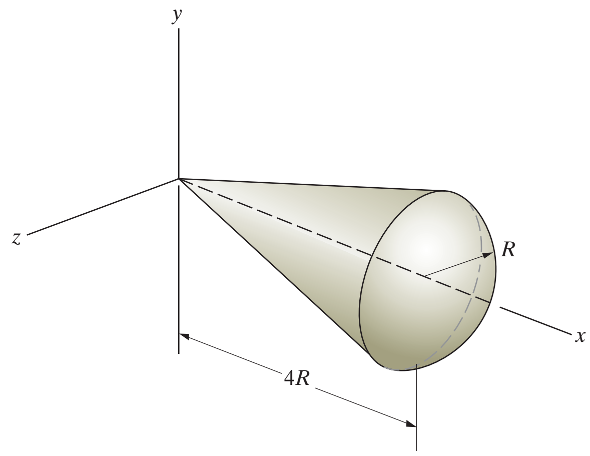
$$\bar{x} = \frac{\bar{x}_1 V_1 + \bar{x}_2 V_2 + \bar{x}_3 V_3}{V_1 + V_2 + V_3}$$

$$= \frac{(30)\ [(60)\ (50)\ (20)] + \left[60 + \frac{4(25)}{3\pi}\right]\left[\frac{1}{2}\pi(25)^2\ (20)\right] + (60)\ \left[-\pi(10)^2(20)\right]}{(60)\ (50)\ (20) + \frac{1}{2}\pi(25)^2\ (20) - \pi(10)^2\ (20)}$$

$= 38.3$ mm.

$\boxed{\bar{x} = 38.3 \text{ mm}}$

Problem 7.73 Determine the centroids of the volumes.



Solution: The object will be divided into a cone and a hemisphere.

From symmetry $\overline{y} = \overline{z} = 0$

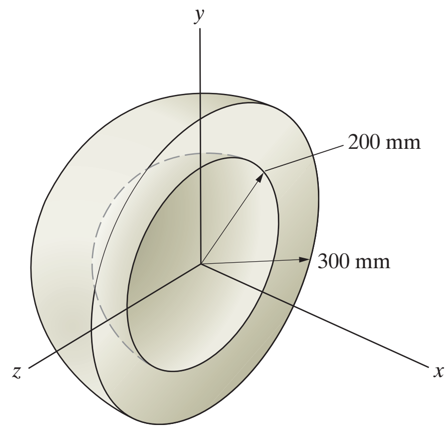
Using tables we have in the x direction

$$\overline{x} = \frac{\frac{3}{4}(4R)\left(\frac{1}{3}\pi R^2[4R]\right) + \left(4R + \frac{3R}{8}\right)\left(\frac{2\pi R^3}{3}\right)}{\frac{1}{3}\pi R^2[4R] + \frac{2\pi R^3}{3}} = \frac{83R}{24}$$

In summary

$$\boxed{\overline{x} = \frac{83R}{24},\ \overline{y} = 0,\ \overline{z} = 0}$$

Problem 7.74 Determine the centroids of the volumes.



Solution: We have a hemisphere and a hemispherical hole. From symmetry $\overline{y} = \overline{z} = 0$

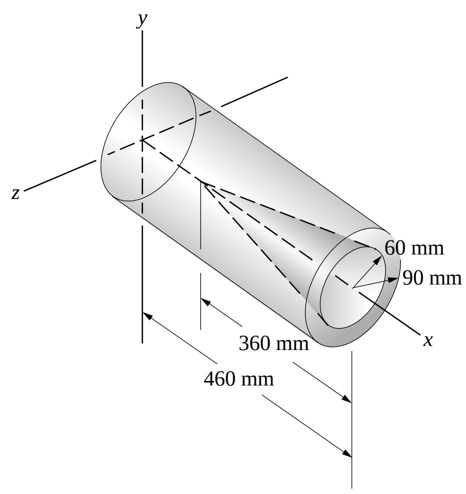
In the x direction we have

$$\overline{x} = \frac{\left(-\frac{3[300\text{ mm}]}{8}\right)\left(\frac{2\pi[300\text{ mm}]^3}{3}\right) - \left(-\frac{3[200\text{ mm}]}{8}\right)\left(\frac{2\pi[200\text{ mm}]^3}{3}\right)}{\frac{2\pi[300\text{ mm}]^3}{3} - \frac{2\pi[200\text{ mm}]^3}{3}}$$

We have

$$\boxed{\overline{x} = -128\text{ mm},\ \overline{y} = 0,\ \overline{z} = 0}$$

Problem 7.75 Determine the centroids of the volumes.



Solution: This is a composite shape. Let us consider a solid cylinder and then subtract the cone. Use information from the appendix

	Volume	Volume (mm^3)	**x**	**x** (mm)
Cylinder	$\pi R^2 L$	1.1706×10^7	$L/2$	230
Cone	$\frac{1}{3}\pi r^2 h$	1.3572×10^6	L-$h/4$	370

$R = 90$ mm

$L = 460$ mm

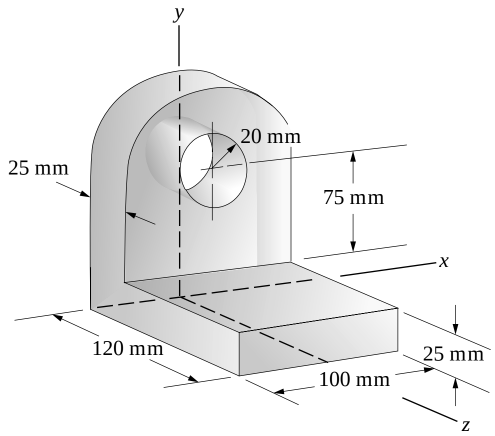
$r = 60$ mm

$h = 360$ mm

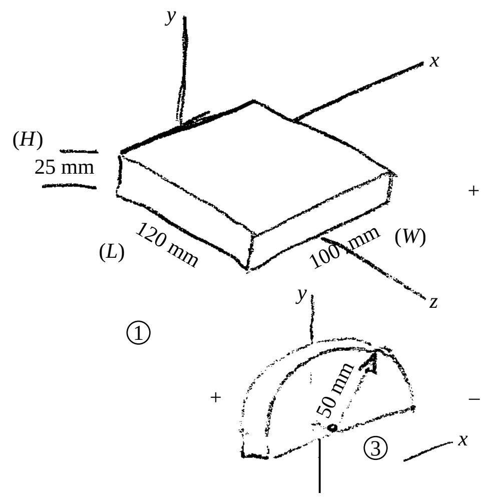
$$\mathbf{x} = \frac{X_{CyL}V_{CyL} - X_{\text{CONE}}V_{\text{CONE}}}{V_{CyL} - V_{\text{CONE}}}$$

$$\boxed{\begin{array}{l}\mathbf{x} = 211.6\text{ mm}\\ \mathbf{y} = \mathbf{z} = 0\text{ mm}\end{array}}$$

Problem 7.76 Determine the centroids of the volumes.



Solution: Break the composite object into simple shapes, find the volumes and centroids of each, and combine to find the required centroid.



Object	Volume (V)	**x**	**y**	**z**
1	LWH	0	$H/2$	$L/2$
2	hWD	0	$(H + h/2)$	$D/2$
3	$\pi R^2 D/2$	0	$\left(H + h + \dfrac{4R}{3\pi}\right)$	$D/2$
4	$\pi r^2 D$	0	$(H + h)$	$D/2$

where $R = W/2$. For the composite,

$$\mathbf{x} = \frac{\mathbf{x}_1 V_1 + \mathbf{x}_2 V_2 + \mathbf{x}_3 V_3 - \mathbf{x}_4 V_4}{V_1 + V_2 + V_3 - V_4}$$

with similar eqns for **y** and **z**

The dimensions, from the figure, are

$L = 120$ mm

$W = 100$ mm

$H = 25$ mm

$r = 20$ mm

$h = 75$ mm

$D = 25$ mm

$R = 50$ mm

Object	V mm^3	**x** (mm)	**y** (mm)	**z** (mm)
+1	300000	0	12.5	60
+2	187500	0	62.5	12.5
+3	98175	0	121.2	12.5
−4	31416	0	100	12.5

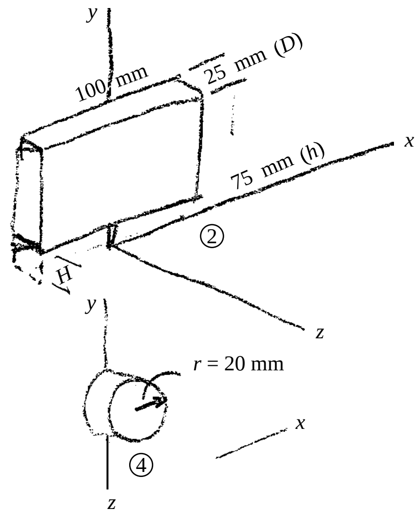
7.76 (*Continued*)

Substituting into the formulas for the composite, we get

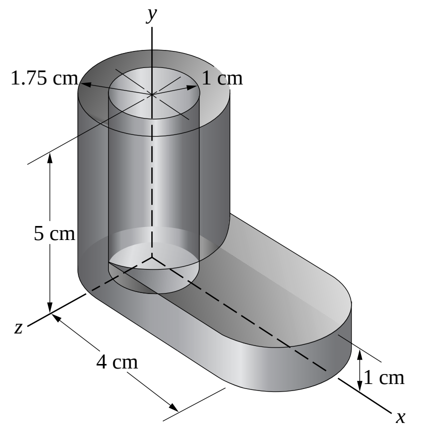
$\mathbf{x} = 0$

$\mathbf{y} = 43.7$ mm

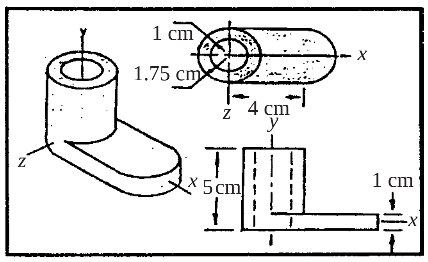
$\mathbf{z} = 38.2$ mm



Problem 7.77 Determine the centroids of the volumes.



Solution: Divide the object into six volumes: (1) A cylinder 5 cm long of radius 1.75 cm, (2) a cylinder 5 cm long of radius 1 cm, (3) a block 4 cm long, 1 cm thick, and 2(1 75) = 3.5 cm wide. (4) Semi-cylinder 1 cm long with a radius of 1.75 cm, (5) a semi-cylinder 1 cm long with a radius of 1.75 cm. (6) The composite object. The volumes and centroids are:



Volume	Vol, cu cm	**x**, cm	**y**, cm	**z**, cm
V1	48.1	0	2.5	0
V2	15.7	0	2.5	0
V3	14	2	0.5	0
V4	4.81	0.743	0.5	0
V5	4.81	0	4.743	0

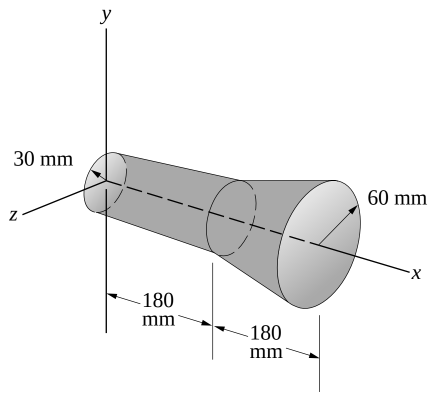
The composite volume is $V = V_1 - V_2 + V_3 - V_4 + V_5 = 46.4$ cm^3. The composite centroid:

$$\mathbf{x} = \frac{V_1\mathbf{x}_1 - V_2\mathbf{x}_2 + V_3\mathbf{x}_3 - V_4\mathbf{x}_4 + V_5\mathbf{x}_5}{V} = 1.02 \text{ cm},$$

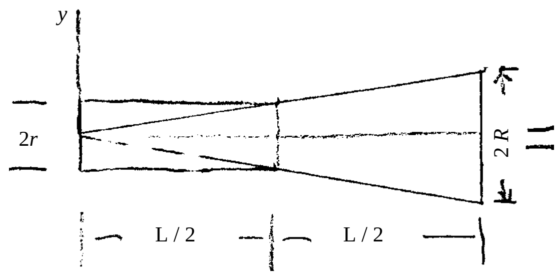
$$\mathbf{y} = \frac{V_1 y_1 - V_2 y_2 + V_3 y_3 - V_4\mathbf{y}_4 + V_5\mathbf{y}_5}{V} = 1.9 \text{ cm},$$

$\mathbf{z} = 0$

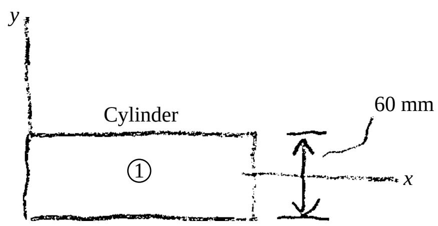
Problem 7.78 Determine the centroids of the volumes.



Solution: Consider the composite volume as being made up of three volumes, a cylinder, a large cone, and a smaller cone which is removed



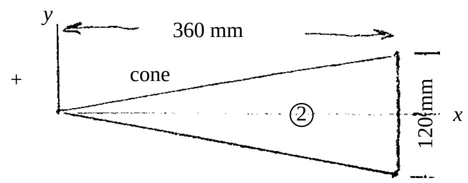
Object	V	$\mathbf{x}$
Cylinder	$\pi r^2 L/2$	$L/4$
Cone 1	$\frac{1}{3}\pi R^2 L$	$3L/4$
Cone 2	$\frac{1}{3}\pi r^2\left(\frac{L}{2}\right)$	$3(L/2)/4$
	(mm^3)	(mm)
Cylinder	5.089×10^5	90
Cone 1	1.357×10^6	270
Cone 2	1.696×10^5	135



$L = 360$ mm

$r = 30$ mm

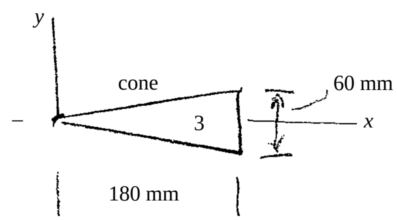
$R = 60$ mm



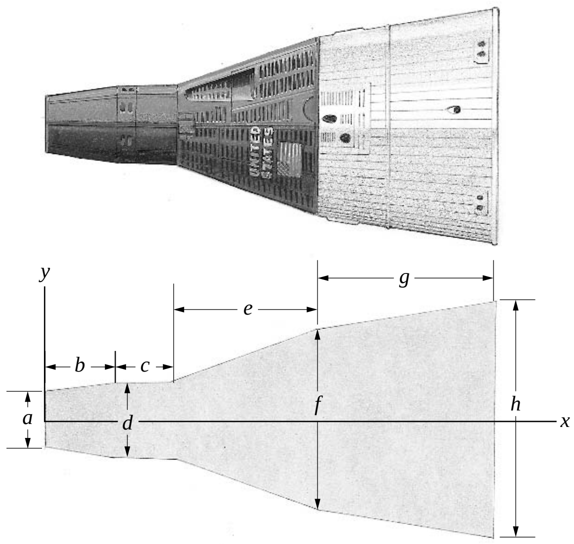
For the composite shape

$$\mathbf{x} = \frac{\mathbf{x}_{Cyl} V_{CyL} + \mathbf{x}_1 V_1 - \mathbf{x}_2 V_2}{V_{CyL} + V_1 - V_2}$$

$\mathbf{x} = 229.5$ mm



Problem 7.79 The dimensions of the *Gemini* spacecraft (in meters) are $a = 0.70$, $b = 0.88$, $c = 0.74$, $d = 0.98$, $e = 1.82$, $f = 2.20$, $g = 2.24$, and $h = 2.98$. Determine the centroid of its volume.



Solution: The spacecraft volume consists of three truncated cones and a cylinder. Consider the truncated cone of length L with radii at the ends R_1 and R_2, where $R_2 > R_1$. Choose the origin of the x–y coordinate system at smaller end. The radius of the cone is a linear function of the length; from geometry, the length of the cone before truncations was

(1) $H = \dfrac{R_2 L}{(R_2 - R_1)}$ with volume

(2) $\dfrac{\pi R_2^2 H}{3}$. The length of the truncated portion is

(3) $\eta = \dfrac{R_1 L}{(R_2 - R_1)}$ with volume

(4) $\dfrac{\pi R_1^2 \eta}{3}$. The volume of the truncated cone is the difference of the two volumes,

(5) $V = \dfrac{\pi L}{3}\left(\dfrac{R_2^3 - R_1^3}{R_2 - R_1}\right)$. The centroid of the removed part of the cone is

(6) $\mathbf{x}_\eta = \left(\dfrac{3}{4}\right)\eta$, and the centroid of the complete cone is

(7) $\mathbf{x}_h = \left(\dfrac{3}{4}\right)H$, measured from the pointed end. From the composite theorem, the centroid of the truncated cone is

(8) $\mathbf{x} = \dfrac{V_h \mathbf{x}_h - V_\eta \mathbf{x}_\eta}{V} - \eta + x$, where x is the x-coordinate of the left hand edge of the truncated cone in the specific coordinate system. These eight equations are the algorithm for the determination of the volumes and centroids of the truncated cones forming the spacecraft.

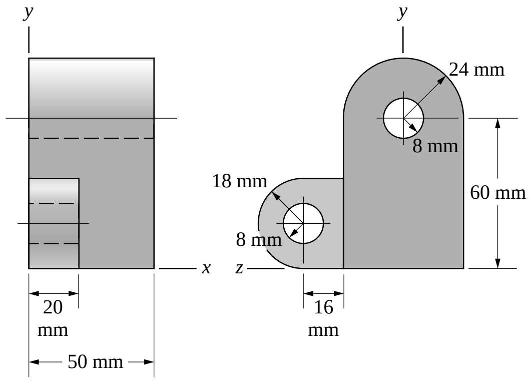
Beginning from the left, the volumes are (1) a truncated cone, (2) a cylinder, (3) a truncated cone, and (4) a truncated cone. The algorithm and the data for these volumes were entered into **TK Solver Plus** and the volumes and centroids determined. The volumes and x-coordinates of the centroids are:

Volume	Vol, cu m	**x**, m
V1	0.4922	0.4884
V2	0.5582	1.25
V3	3.7910	2.752
V4	11.8907	4.8716
Composite	16.732	3.999

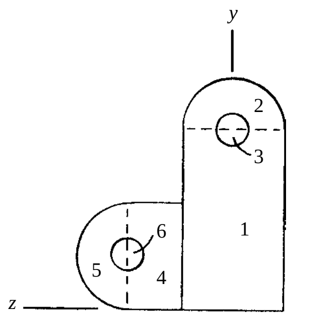
The last row is the composite volume and x-coordinate of the centroid of the composite volume.

The total length of the spacecraft is 5.68 m, so the centroid of the volume lies at about 69% of the length as measured from the left end of the spacecraft. *Discussion:* The algorithm for determining the centroid of a system of truncated cones may be readily understood if it is implemented for a cone of known dimensions divided into sections, and the results compared with the known answer. Alternate algorithms (e.g. a Pappus-Guldinus algorithm) are useful for checking but arguably do not simplify the computations *End discussion.*

Problem 7.80 Two views of a machine element are shown. Determine the centroid of its volume.



Solution: We divide the volume into six parts as shown. Parts 3 and 6 are the "holes", which each have a radius of 8 mm. The volumes are



$V_1 = (60)(48)(50) = 144{,}000 \text{ mm}^3,$

$V_2 = \frac{1}{2}\Pi(24)^2(50) = 45, 239 \text{ mm}^3,$

$V_3 = \Pi(8)^2(50) = 10, 053 \text{ mm}^3,$

$V_4 = (16)(36)(20) = 11, 520 \text{ mm}^3,$

$V_5 = \frac{1}{2}\Pi(18)^2(20) = 10, 179 \text{ mm}^3,$

$V_6 = \Pi(8)^2(20) = 4021 \text{ mm}^3.$

The coordinates of the centroids are

$\mathbf{x}_1 = 25$ mm,

$\mathbf{y}_1 = 30$ mm,

$\mathbf{z}_1 = 0,$

$\mathbf{x}_2 = 25$ mm,

$$\mathbf{y}_2 = 60 + \frac{4(24)}{3\Pi} = 70.2 \text{ mm},$$

$\mathbf{z}_2 = 0,$

$\mathbf{x}_3 = 25$ mm,

$\mathbf{y}_3 = 60$ mm,

$\mathbf{z}_3 = 0,$

$\mathbf{x}_4 = 10$ mm,

$\mathbf{y}_4 = 18$ mm,

$\mathbf{z}_4 = 24 + 8 = 32$ mm,

$\mathbf{x}_5 = 10$ mm,

$\mathbf{y}_5 = 18$ mm,

$$\mathbf{z}_5 = 24 + 16 + \frac{4(18)}{3\Pi} = 47.6 \text{ mm},$$

$\mathbf{x}_6 = 10$ mm,

$\mathbf{y}_6 = 18$ mm,

$\mathbf{z}_6 = 24 + 16 = 40$ mm.

The x coordinate of the centroid is

$$\mathbf{x} = \frac{\mathbf{x}_1 V_1 + \mathbf{x}_2 V_2 - \mathbf{x}_3 V_3 + \mathbf{x}_4 V_4 + \mathbf{x}_5 V_5 - \mathbf{x}_6 V_6}{V_1 + V_2 - V_3 + V_4 + V_5 - V_6} = 23.65 \text{ mm}.$$

Calculating the y and z coordinates in the same way, we obtain $y =$ 36.63 mm and $z = 3.52$ mm

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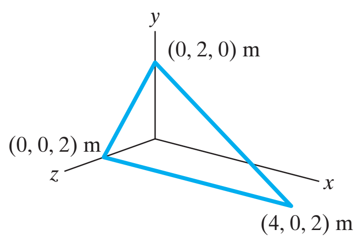
Problem 7.81 In Example 7.13, suppose that the circular arc is replaced by a straight line as shown. Determine the centroid of the three-segment line.



Solution: Let the new straight-line segment be line 1 and let the segment in the x-z plane be line 2. Let the other line segment be line 3. The centroid locations of the parts and their lengths are

$\bar{x}_1 = 0,\quad \bar{y}_1 = 1 \text{ m},\quad \bar{z}_1 = 1 \text{ m},\quad L_1 = 2.83 \text{ m},$

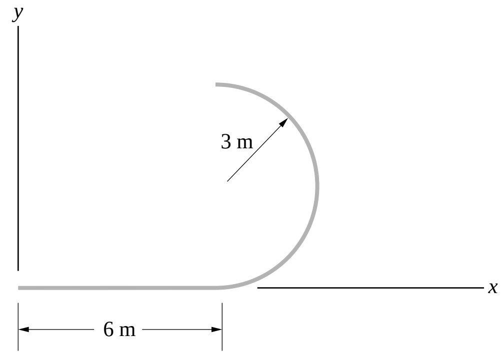
$\bar{x}_2 = 2 \text{ m},\quad \bar{y}_2 = 1 \text{ m},\quad \bar{z}_2 = 2 \text{ m},\quad \text{L}_2 = 4 \text{ m},$

$\bar{x}_3 = 2 \text{ m},\quad \bar{y}_3 = 1 \text{ m},\quad \bar{z}_3 = 1 \text{ m},\quad L_2 = 4.90 \text{ m}.$

Applying Eqs. (7.18) yields

$$\boxed{\bar{x} = 1.52 \text{ m},\quad \bar{y} = 0.659 \text{ m},\quad \bar{z} = 1.34 \text{ m}}$$

Problem 7.82 Determine the centroids of the lines.



Solution: The object is divided into two lines and a composite.

(1) $L_1 = 6$ m, $\mathbf{x}_1 = 3$ m, $\mathbf{y}_1 = 0$.

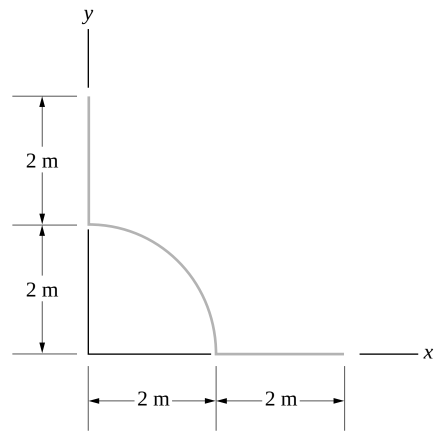
(2) $L_2 = 3\pi$ m, $\mathbf{x}_2 = 6 + \dfrac{6}{\pi}$ m (*Note*: See Example 7.13) $\mathbf{y}_2 = 3$.

(3) The composite length: $L = 6 + 3\pi$ m. The composite centroid:

$$\mathbf{x} = \frac{L_1\mathbf{x}_1 + L_2\mathbf{x}_2}{L} = 6 \text{ m},$$

$$\mathbf{y} = \frac{3\pi}{2+\pi} = 1.83 \text{ m}$$

Problem 7.83 Determine the centroids of the lines.



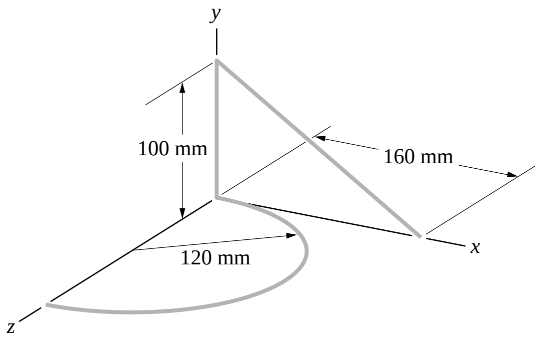
Solution: Break the composite line into three parts (the quarter circle and two straight line segments) (see Appendix B).

	$\mathbf{x}_i$	$\mathbf{y}_i$	L_i	
Part 1	$2R/\pi$	$2R/\pi$	$\pi R/2$	($R = 2$ m)
Part 2	3 m	0	2 m	
Part 3	0	3 m	2 m	

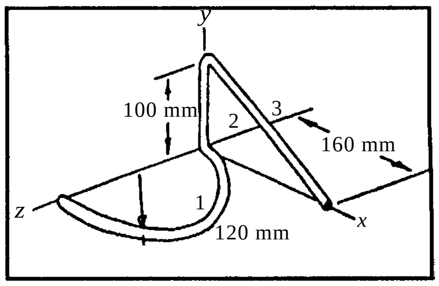
$$\overline{x} = \frac{x_1L_1 + x_2L_2 + x_3L_3}{L_1 + L_2 + L_3} = 1.4 \text{ m}$$

$$\overline{y} = \frac{y_1L_1 + y_2L_2 + y_3L_3}{L_1 + L_2 + L_3} = 1.4 \text{ m}$$

Problem 7.84 The semicircular part of the line lies in the x–z plane. Determine the centroid of the line.



Solution: The bar is divided into three segments plus the composite. The lengths and the centroids are given in the table: The composite length is:



$$L = \sum_{i=1}^{3} L_i.$$

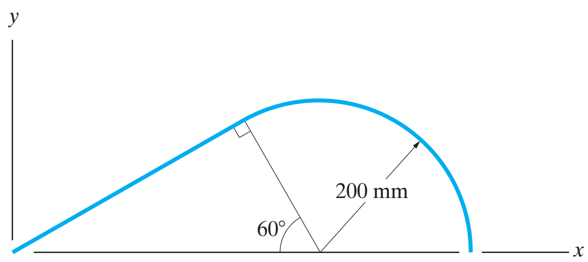
The composite coordinates are:

$$\mathbf{x} = \frac{\sum_{i=1}^{3} L_i \mathbf{x}_i}{L},$$

and $\mathbf{y} = \dfrac{\sum_{i=1}^{3} L_i \mathbf{y}_i}{L}$

Segment	Length, mm	**x**, mm	**y**, mm	**z**, mm
L1	120π	$\frac{240}{\pi}$	0	120
L2	100	0	50	0
L3	188.7	80	50	0
Composite	665.7	65.9	21.7	68.0

Problem 7.85 Determine the centroid of the line.



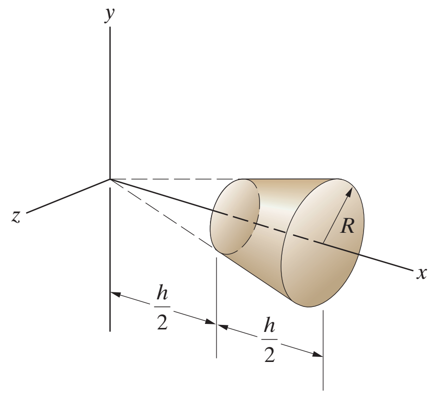
Solution: Break into a straight line and an arc.

$$\bar{x} = \frac{\frac{1}{2}(200 \text{ mm} \tan 60^\circ)^2 \cos 30^\circ + \int_0^{2\pi/3} (200 \text{ mm})^2 \left(\frac{1}{\cos 60^\circ} + \cos\theta\right) d\theta}{200 \text{ mm} \tan 60^\circ + \int_0^{2\pi/3} (200 \text{ mm})\, d\theta} = 332 \text{ mm}$$

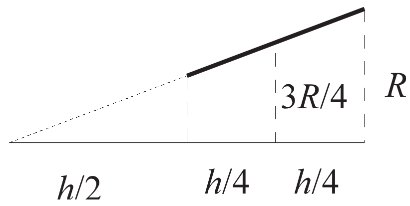
$$\bar{y} = \frac{\frac{1}{2}(200 \text{ mm} \tan 60^\circ)(200 \text{ mm} \sin 60^\circ) + \int_0^{2\pi/3} (200 \text{ mm})^2 (\sin\theta)\, d\theta}{200 \text{ mm} \tan 60^\circ + \int_0^{2\pi/3} (200 \text{ mm})\, d\theta} = 118 \text{ mm}$$

$$\boxed{\bar{x} = 332 \text{ mm}, \quad \bar{y} = 118 \text{ mm}}$$

Problem 7.86 Use the method described in Active Example 7.14 to determine the area of the curved part of the surface of the truncated cone.



Solution: Work with the solid line shown. The surface area is given by

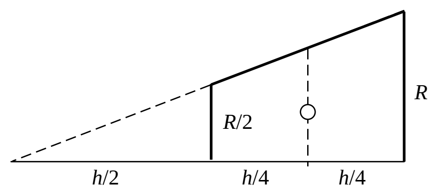


$$A = 2\pi \bar{y} L = 2\pi\left(\frac{3R}{4}\right)\sqrt{\left(\frac{R}{2}\right)^2 + \left(\frac{h}{2}\right)^2}$$

$$\boxed{A = \frac{3\pi R}{4}\sqrt{h^2 + R^2}}$$

Problem 7.87 Use the second Pappus–Guldinus theorem to determine the volume of the truncated cone.

Solution: Work with the trapezoidal area



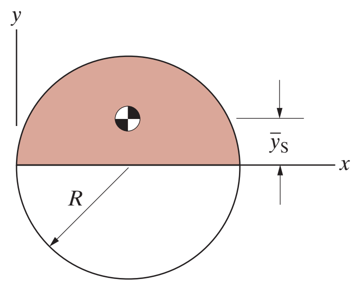
$$A = (R/2)(h/2) + (1/2)(R/2)(h/2) = \frac{3Rh}{8}$$

$$\bar{y} = \frac{(R/2)(h/2)(R/4) + (1/2)(R/2)(h/2)[(1/3)(R/2) + (R/2)]}{A} = \frac{7R}{18}$$

$$V = 2\pi \bar{y} A = 2\pi\left(\frac{7R}{18}\right)\left(\frac{3Rh}{8}\right) = \frac{7\pi R^2 h}{24}$$

$$\boxed{V = \frac{7\pi R^2 h}{24}}$$

Problem 7.88 The area of the shaded semicircle is $\frac{1}{2}\pi R^2$. The volume of a sphere is $\frac{4}{3}\pi R^3$. Extend the approach described in Example 7.15 to the second Pappus–Guldinus theorem and determine the centroid $\bar{y}_S$ of the semicircular area.



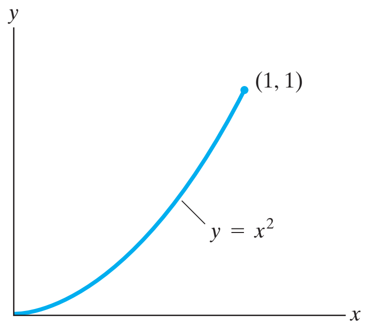
Solution: The semicrcular area is $A = \frac{1}{2}\pi R^2$, and $\bar{y}_s$ is the y coordinate of its centroid. Rotating the area about the x axis generates the volume of a sphere. The second Pappus–Guldinus theorem states that the volume of the sphere is

$$V = 2\pi \bar{y}_s A$$

$$\frac{4}{3}\pi R^3 = 2\pi \bar{y}_s \left(\frac{1}{2}\pi R^2\right)$$

Solving for $\bar{y}_s$ yields $\boxed{\bar{y}_s = \frac{4R}{3\pi}}$

Problem 7.89 Use the second Pappus–Guldinus theorem to determine the volume generated by revolving the curve about the y axis.



Solution: *The x coordinate of the centroid:*. The element of area is the vertical strip of height $(1-y)$ and width dx. Thus

$$A = \int_0^1 (1-y)\,dx = \int_0^1 (1-x^2)\,dx.$$

Integrating,

$$A = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{2}{3}.$$

$$\int_A x\,dA = \int_0^1 (x - x^3)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{4},$$

divide by the area: $\mathbf{x} = \frac{3}{8}$. The volume is $V = 2\pi\mathbf{x}A = \frac{\pi}{2}$

Problem 7.90 The length of the curve is $L = 1.479$, and the area generated by rotating it about the x axis is $A = 3.810$. Use the first Pappus–Guldinus theorem to determine the y coordinate of the centroid of the curve.

Solution: The surface area is $A = 2\pi\mathbf{y}L$, from which

$$\mathbf{y} = \frac{A}{2\pi L} = 0.41$$

Problem 7.91 Use the first Pappus–Guldinus theorem to determine the area of the surface generated by revolving the curve about the y axis.

Solution: The length of the line is given in Problem 7.90. $L = 1.479$. The elementary length of the curve is

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx.$$

Noting $\frac{dy}{dx} = 2x$, the element of line is $dL = (1+4x^2)^{1/2}$.

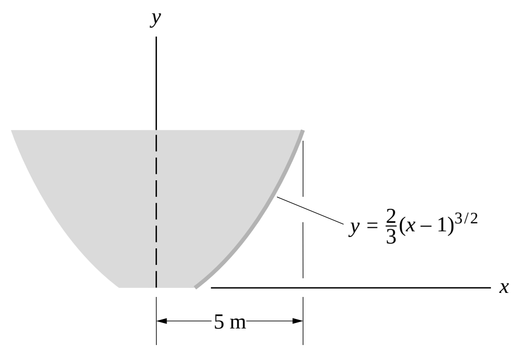
The x-coordinate:

$$\int_L x\,dL = \int_0^1 x(1+4x^2)^{1/2}\,dx$$

$$= \frac{1}{12}\left[(1+4x^2)^{3/2}\right]_0^1 = \frac{5^{3/2}-1}{12} = 0.8484.$$

Divide by the length to obtain $\mathbf{x} = 0.5736$. The surface area is $A = 2\pi\mathbf{x}L = 5.33$

Problem 7.92 A nozzle for a large rocket engine is designed by revolving the function $y = \frac{2}{3}(x-1)^{3/2}$ about the y axis. Use the first Pappus–Guldinus theorem to determine the surface area of the nozzle.



Solution: *The length:* Noting that $\frac{dy}{dx} = (x-1)^{1/2}$, the element of length is

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{x}\, dx$$

from which

$$L = \int_L dL = \int_1^5 (x)^{1/2}\, dx = \left[\frac{2}{3}(x)^{3/2}\right]_1^5 = 6.7869 \text{ m}$$

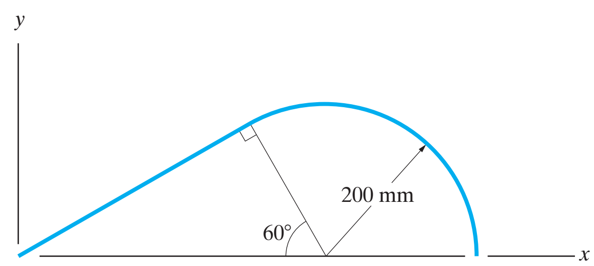
The x-coordinate:

$$\int_L x\, dL = \int_1^5 x^{3/2}\, dx = \left[\frac{2}{5}x^{5/2}\right]_1^5 = 21.961.$$

Divide by the length: $\mathbf{x} = 3.2357$. The area

$$A = 2\pi \mathbf{x} L = 138 \text{ m}^2$$

Problem 7.93 The coordinates of the centroid of the line are $\overline{x} = 332$ mm and $\overline{y} = 118$ mm. Use the first Pappus-Guldinus theorem to determine the area of the surface of revolution obtained by revolving the line about the x axis.



Solution:

$$L = 200 \text{ mm} \tan 60^\circ + 200 \text{ mm}\left(\frac{120^\circ}{180^\circ}\right)\pi = 765 \text{ mm}$$

$$\boxed{A = 2\pi\overline{y}L = 2\pi(0.118 \text{ m})(0.765 \text{ m}) = 0.567 \text{ m}^2}$$

Problem 7.94 The coordinates of the centroid of the area between the x axis and the line in Problem 7.93 are $\overline{x} = 355$ mm and $\overline{y} = 78.4$ mm. Use the second Pappus-Guldinus theorem to determine the volume obtained by revolving the area about the x axis.

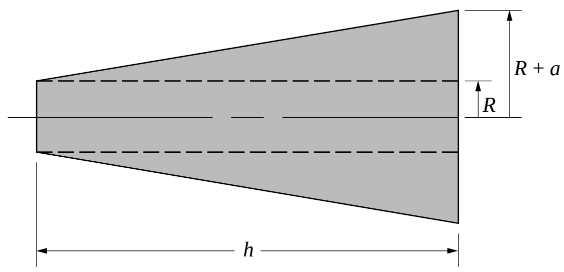
Solution: The area is

$$A = \frac{1}{2}(0.2\text{ m})(0.2\text{ m}\tan 60^\circ) + \left(\frac{120^\circ}{360^\circ}\pi\right)(0.2\text{ m})^2 = .0765\text{ m}^2$$

$$\boxed{V = 2\pi\overline{y}A = 2\pi(0.0784\text{ m})(0.0765\text{ m}^2) = 0.0377\text{ m}^3}$$

Problem 7.95 The volume of revolution contains a hole of radius R.

(a) Use integration to determine its volume.
(b) Use the second Pappus–Guldinus theorem to determine its volume.



Solution:

(a) The element of volume is a disk of radius y and thickness dx. The area of the disk is $\pi(y^2 - R^2)$. The radius is

$$y = \left(\frac{a}{h}\right)x + R,$$

from which $dV = \pi\left(\frac{a}{h}x + R\right)^2 dx - \pi R^2\,dx.$

Denote $m = \left(\frac{a}{h}\right)$, $dV = \pi(m^2x^2 + 2mRx)\,dx,$

from which

$$V = \int_V dV = \pi m\int_0^h (mx^2 + 2Rx)\,dx$$

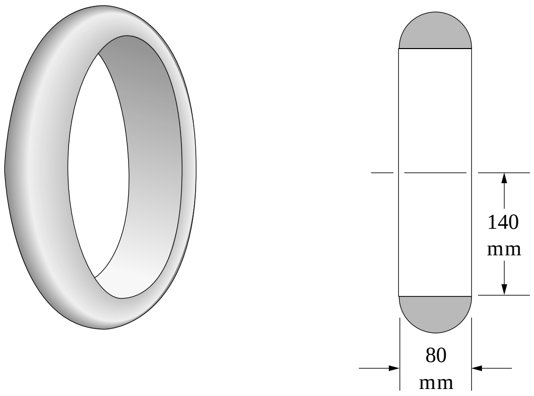
$$= \pi m\left[m\frac{x^3}{3} + Rx^2\right]_0^h = \pi mh^2\left(\frac{mh}{3} + R\right)$$

$$= \pi ah\left(\frac{a}{3} + R\right).$$

(b) The area of the triangle is $A = (\frac{1}{2})ah$. The y-coordinate of the centroid is $\mathbf{y} = R + (\frac{1}{3})a$. The volume is

$$\boxed{V = 2\pi\mathbf{y}A = \pi ah(R + (\tfrac{1}{3})a)}$$

Problem 7.96 Determine the volume of the volume of revolution.



Solution: The area of the semicircle is $A = \frac{\pi r^2}{2}$. The centroid is $y = R + \frac{4r}{3\pi}$. The volume is

$$V = 2\pi \left(\frac{\pi r^2}{2}\right)\left(R + \frac{4r}{3\pi}\right) = \pi^2 r^2 \left(R + \frac{4r}{3\pi}\right).$$

For $r = 40$ mm and $R = 140$ mm, $\boxed{V = 2.48 \times 10^{-3}\ \text{m}^3}$

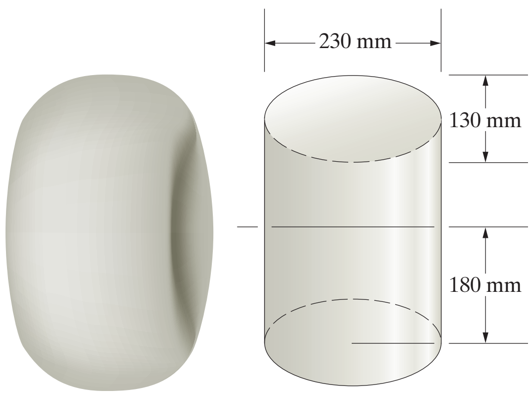
Problem 7.97 Determine the surface area of the volume of revolution in Problem 7.96.

Solution: The length and centroid of the semicircle is $L_o = \pi r$, $\mathbf{y} = R + \frac{2r}{\pi}$. The length and centroid of the inner line is $L_i = 2r$, and $\mathbf{y} = R$.

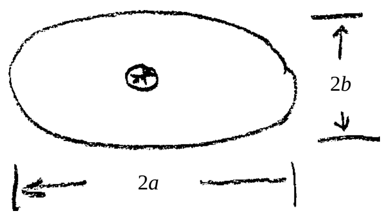
$$A = 2\pi(\pi r)\left(R + \frac{2r}{\pi}\right) + 2\pi(2r)(R) = 2\pi r(\pi R + 2r + 2R).$$

For $r = 40$ mm and $R = 140$ mm, $A = 0.201\ \text{m}^2$

Problem 7.98 The volume of revolution has an elliptical cross section. Determine its volume.



Solution: Use the second theorem of Pappus-Guldinus. The centroid of the ellipse is 180 mm from the axis of rotation. The area of the ellipse is πab where $a = 115$ mm, $b = 65$ mm.

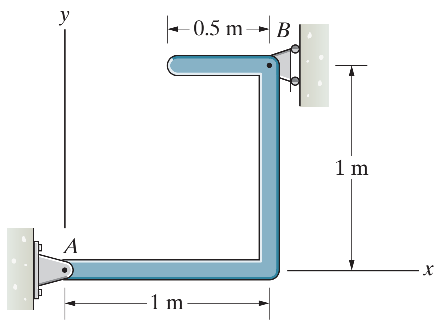


The centroid moves through a distance $|d| = 2\pi R = 2\pi$ (180 mm) as the ellipse is rotated about the axis.

$V = Ad = \pi abd = 2.66 \times 10^7\ \text{mm}^3$

$v = 0.0266\ \text{m}^3$

Problem 7.99 Suppose that the bar in Active Example 7.16 is replace with this 100-kg homogeneous bar. (a) What is the x coordinate of the bar's center of mass? (b) Determine the reactions at A and B.

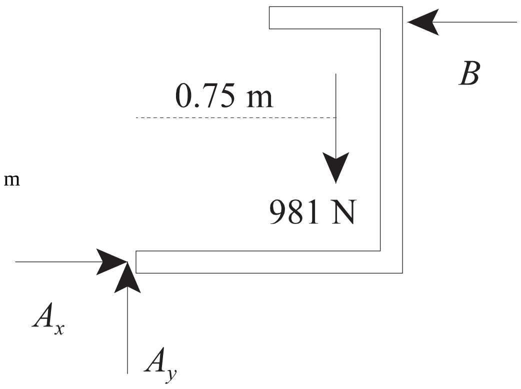


Solution:

(a) Let the new horizontal segment of the bar part 3. The x coordinate of the centroid of the bar's axis, which coincides with its center of mass is

$$\bar{x} = \frac{\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3}{L_1 + L_2 + L_3} = \frac{(0.5)(1) + (1)(1) + (0.75)(0.5)}{0.5 + 1 + 0.5} = 0.75 \text{ m}$$

$\boxed{\bar{x} = 0.75 \text{ m}}$



(b) The equilibrium equations are

$\Sigma M_A : B(1 \text{ m}) - (981 \text{ N})\,(0.75 \text{ m}) = 0,$

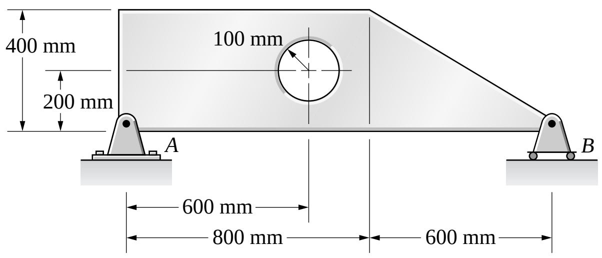
$\Sigma F_x : A_x - B = 0,$

$\Sigma F_y : A_y - 981 \text{ N} = 0.$

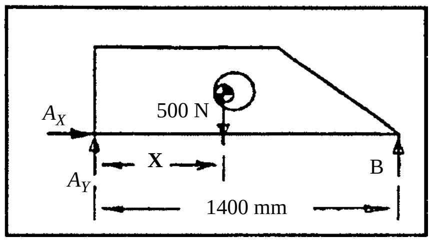
Solving yields

$\boxed{A_x = 736 \text{ N}, \quad A_y = 981 \text{ N}, \quad B = 736 \text{ N}.}$

Problem 7.100 The mass of the homogeneous flat plate is 50 kg. Determine the reactions at the supports *A* and *B*.



Solution: Divide the object into three areas and the composite. Since the distance to the action line of the weight is the only item of importance, and since there is no horizontal component of the weight, it is unnecessary to determine any centroid coordinate other than the x-coordinate. The areas and the x-coordinate of the centroid are tabulated. The last row is the composite area and x-coordinate of the centroid.



Area	A, sq mm	x
Rectangle	3.2×10^5	400
Circle	3.14×10^4	600
Triangle	1.2×10^5	1000
Composite	4.09×10^5	561

The composite area is $A = A_{\text{rect}} - A_{\text{circ}} + A_{\text{triang}}$. The composite x-coordinate of the centroid is

$$\mathbf{x} = \frac{A_{\text{rect}}\mathbf{x}_{\text{rect}} - A_{\text{circ}}\mathbf{x}_{\text{circ}} + A_{\text{triang}}\mathbf{x}_{\text{triang}}}{A}.$$

The sum of the moments about A:

$$\sum M_A = -500(561) + 1400B = 0,$$

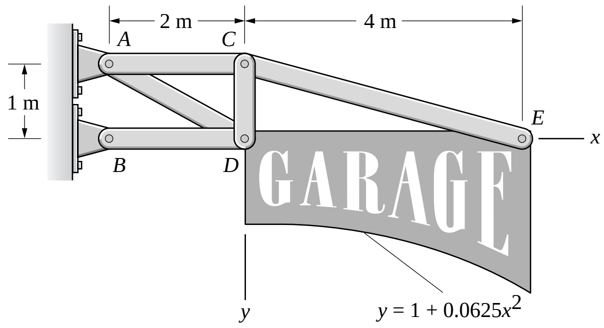
from which $B = 200$ N. The sum of the forces:

$$\sum F_y = A_y + B - 500 = 0,$$

from which $A_y = 300$ N.

$$\sum F_x = A_x = 0$$

Problem 7.101 The suspended sign is a homogeneous flat plate that has a mass of 130 kg. Determine the axial forces in members AD and CE. (Notice that the y axis is positive downward.)



Solution: The strategy is to determine the distance to the action line of the weight (x-coordinate of the centroid) from which to apply the equilibrium conditions to the method of sections.

The area: The element of area is the vertical strip of length y and width dx. The element of area $dA = y\,dx = (1 + ax^2)\,dx$, where $a = 0.0625$. Thus

$$A = \int_A dA = \int_0^4 (1 + ax^2)\,dx = \left[x + \frac{ax^3}{3}\right]_0^4 = 5.3333 \text{ sq ft.}$$

The x-coordinate:

$$\int_A x\,dA = \int_0^4 x(1 + ax^2)\,dx = \left[\frac{x^2}{2} + \frac{ax^4}{4}\right]_0^4 = 12.$$

Divide A: $\mathbf{x} = \dfrac{12}{5.3333} = 2.25$ ft.

The equilibrium conditions: The angle of the member CE is

$\alpha = \tan^{-1}(\frac{1}{4}) = 14.04°$.

The weight of the sign is $W = 130(9.81) = 1275.3$ N. The sum of the moments about D is

$$\sum M_D = -2.25W + 4CE\sin\alpha = 0,$$

from which $\boxed{CE = 2957.7 \text{ N } (T)}$.

Method of sections: Make a cut through members AC, AD and BD and consider the section to the right. The angle of member AD is

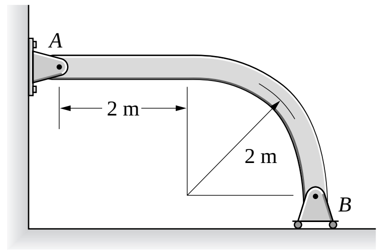
$\beta = \tan^{-1}(\frac{1}{2}) = 26.57°$.

The section as a free body: The sum of the vertical forces:

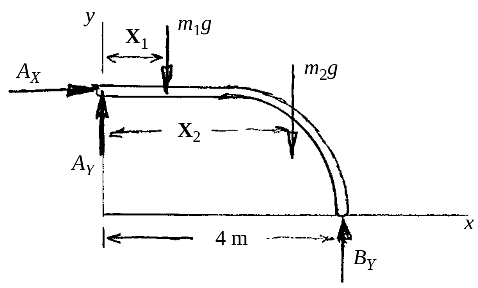
$$\sum F_Y = AD\sin\beta - W = 0$$

from which $\boxed{AD = 2851.7 \text{ N } (T)}$

Problem 7.102 The bar has a mass of 80 kg. What are the reactions at A and B?



Solution: Break the bar into two parts and find the masses and centers of masses of the two parts. The length of the bar is



$L = L_1 + L_2 = 2\text{ m} + 2\pi R/4 (R = 2\text{ m})$

$L = 2 + \pi\text{ m}$

Part	Length_i (m)	Mass_i (kg)	$\mathbf{x}_i$ (m)
1	2	$\left(\dfrac{2}{2+\pi}\right)80$	1
2	π	$\left(\dfrac{\pi}{2+\pi}\right)80$	$\left(2+\dfrac{2R}{\pi}\right)$

$m_1 = 31.12\text{ kg}\quad \mathbf{x}_1 = 1\text{ m}$

$m_2 = 48.88\text{ kg}\quad \mathbf{x}_2 = 3.27\text{ m}$

$\sum F_x:\quad A_x = 0$

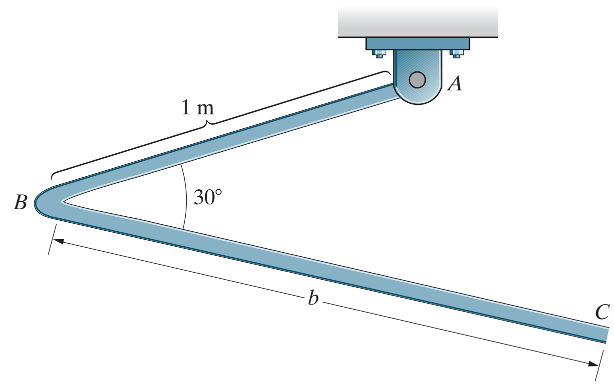
$\sum F_y:\quad A_y + B_y - m_1 g - m_2 g = 0$

$\sum M_A:\quad -\mathbf{x}_1 m_1 g - \mathbf{x}_2 m_2 g + 4B_y = 0$

Solving

$A_x = 0,\ A_y = 316\text{ N},\ B = 469\text{ N}$

Problem 7.103 The mass of the bar per unit length is 2 kg/m. Choose the dimension b so that part BC of the suspended bar is horizontal. What is the dimension b, and what are the resulting reactions on the bar at A?

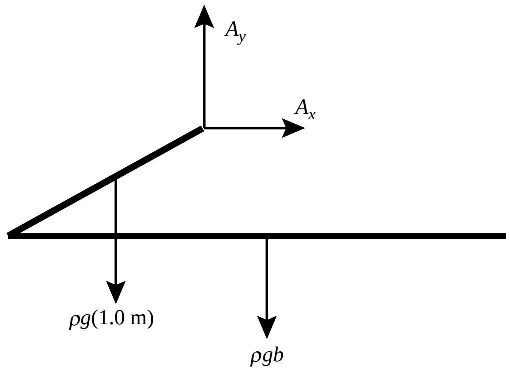


Solution: We must have

$$\sum M_A : \rho g(1.0\text{ m})(0.5\text{ m}\cos 30^\circ)$$

$$-\rho g b\left(\frac{b}{2} - 1.0\text{ m}\cos 30^\circ\right) = 0$$

$$\Rightarrow b = 2.14\text{ m}$$



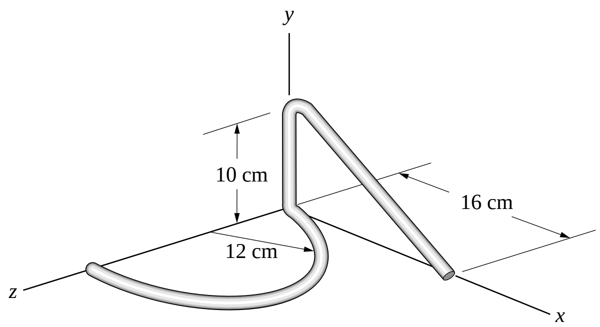
Then

$$\sum F_x : A_x = 0$$

$$\sum F_y : A_y - \rho g(1.0\text{ m}) - \rho g(b) = 0 \quad \Rightarrow$$

$$\boxed{A_x = 0,\ A_y = 61.6\text{ N},\ b = 2.14\text{ m}}$$

Problem 7.104 The semicircular part of the homogeneous slender bar lies in the x–z plane. Determine the center of mass of the bar.



Solution: The bar is divided into three segments plus the composite. The lengths and the centroids are given in the table: The composite length is:

$$L = \sum_{i=1}^{3} L_i.$$

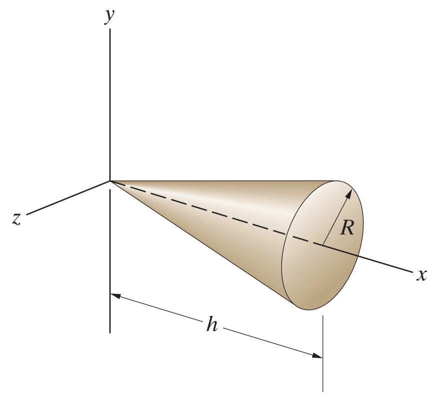
The composite coordinates are:

$$\mathbf{x} = \frac{\sum_{i=1}^{3} L_i \mathbf{x}_i}{L},$$

and $$\mathbf{y} = \frac{\sum_{i=1}^{3} L_i \mathbf{y}_i}{L}$$

Segment	Length, cm	**x**, cm	**y**, cm	**z**, cm
L1	12π	$\frac{24}{\pi}$	0	12
L2	10	0	5	0
L3	18.868	8	5	0
Composite	66.567	6.594	2.168	6.796

Problem 7.105 The density of the cone is given by the equation $\rho = \rho_0(1 + x/h)$, where ρ_0 is a constant. Use the procedure described in Example 7.17 to show that the mass of the cone is given by $m = (7/4)\rho_0 V$, where V is the volume of the cone, and that the x coordinate of the center of mass of the cone is $\bar{x} = (27/35)h$.



Solution: Consider an element of volume dV of the cone in the form of a "disk" of width dx. The radius of such a disk at position x is $(R/h)x$, so $dV = \pi[(R/h)x]^2\ dx$.

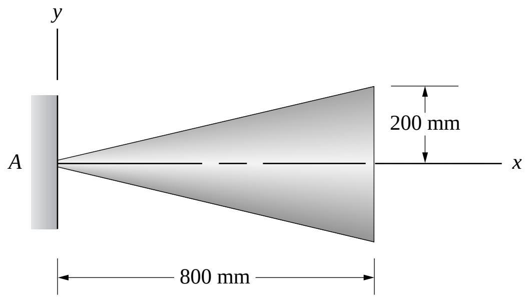
The mass of the cone is

$$m = \int_V \rho dV = \int_0^h \rho_0\ (1 + x/h)\ \pi\ [(R/h)x]^2\ dx = \frac{7}{12}\ \rho_0 \pi R^2 h = \frac{7}{3}\ \rho_0 V.$$

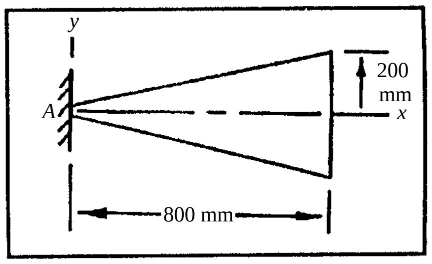
The x coordinate of the center of mass is

$$\bar{x} = \frac{\int_V x\rho dV}{\int_V \rho dV} = \frac{\int_0^h x\rho_0\ (1 + x/h)\ \pi\ [(R/h)x]^2\ dx}{\int_0^h \rho_0\ (1 + x/h)\ \pi\ [(R/h)\ x]^2\ dx} = \frac{27}{35}\ h.$$

Problem 7.106 A horizontal cone with 800-mm length and 200-mm radius has a built-in support at A. Its density is $\rho = 6000(1 + 0.4x^2)$ kg/m^3, where x is in meters. What are the reactions at A?



Solution: The strategy is to determine the distance to the line of action of the weight, from which to apply the equilibrium conditions.



The mass: The element of volume is a disk of radius y and thickness dx. y varies linearly with x: $y = 0.25x$. Denote $a = 0.4$. The mass of the disk is

$$dm = \rho\pi y^2\,dx = 6000\pi(1 + ax^2)(0.25x)^2\,dx$$

$$= 375\pi(1 + ax^2)x^2\,dx,$$

from which

$$m = 375\pi\int_0^{0.8}(1 + ax^2)x^2\,dx = 375\pi\left[\frac{x^3}{3} + a\frac{x^5}{5}\right]_0^{0.8}$$

$$= 231.95\text{ kg}$$

The x-coordinate of the mass center:

$$\int_m x\,dm = 375\pi\int_0^{0.8}(1 + ax^2)x^3\,dx = 375\pi\left[\frac{x^4}{4} + a\frac{x^6}{6}\right]_0^{0.8}$$

$$= 141.23.$$

Divide by the mass: $\mathbf{x} = 0.6089$ m
The equilibrium conditions: The sum of the moments about A:

$$\sum M = M_A - mg\mathbf{x} = 0,$$

from which

$$M_A = mg\mathbf{x} = 231.94(9.81)(0.6089)$$

$$\boxed{= 1385.4\text{ N-m}}.$$

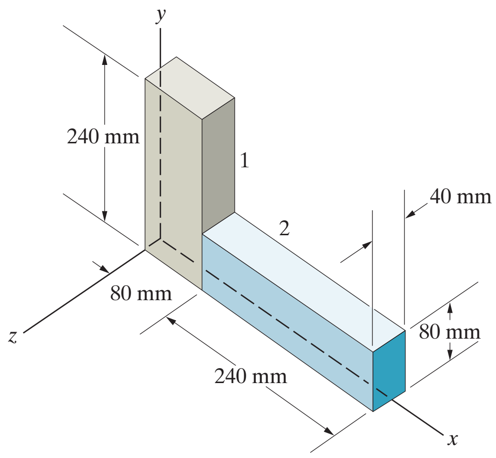
The sum of the vertical forces:

$$\sum F_Y = A_Y - mg = 0$$

from which $\boxed{A_Y = 2275.4\text{ N}}$. The horizontal component of the reaction is zero,

$$\sum F_X = 0.$$

Problem 7.107 In Active Example 7.18, suppose that bar 1 is replaced by a bar with the same dimensions that consists of aluminum alloy with a density of 2600 kg/m^3. Determine the x coordinate of the center of mass of the machine part.



Solution: The mass of bar 1 is

$$m = (7.68 \times 10^{-4}\ \text{m}^3)\ (2600\ \text{kg/m}^3)$$

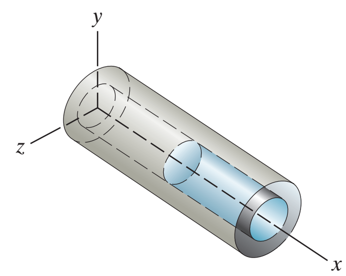
$$= 2.00\ \text{kg}$$

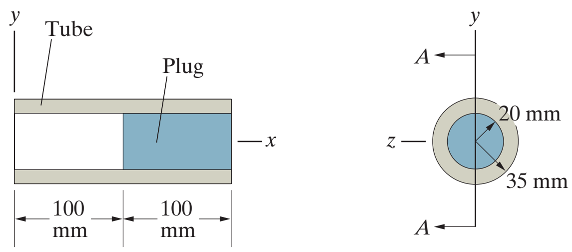
The x coordinate of the center of mass is

$$\bar{x} = \frac{\bar{x}_1 m_1 + \bar{x}_2 m_2}{m_1 + m_2} = \frac{(40\ \text{mm})\ (2.00\ \text{kg}) + (200\ \text{mm})\ (5.99\ \text{kg})}{2.00\ \text{kg} + 5.99\ \text{kg}} = 160\ \text{mm}$$

$$\boxed{\bar{x} = 160\ \text{mm}}$$

Problem 7.108 The cylindrical tube is made of aluminum with mass density 2700 kg/m^3. The cylindrical plug is made of steel with mass density 7800 kg/m^3. Determine the coordinates of the center of mass of the composite object.





Section A-A

Solution: The volume of the aluminum tube is

$$V_{Al} = \pi(0.035^2 - 0.02^2)(0.2) = 5.18 \times 10^{-4}\ \text{m}^3.$$

The mass of the aluminum tube is $m_{Al} = (2700)V_{Al} = 1.4$ kg. The centroid of the aluminum tube is $\mathbf{x}_{AL} = 0.1$ m, $\mathbf{y}_{Al} = \mathbf{z}_{Al} = 0$.

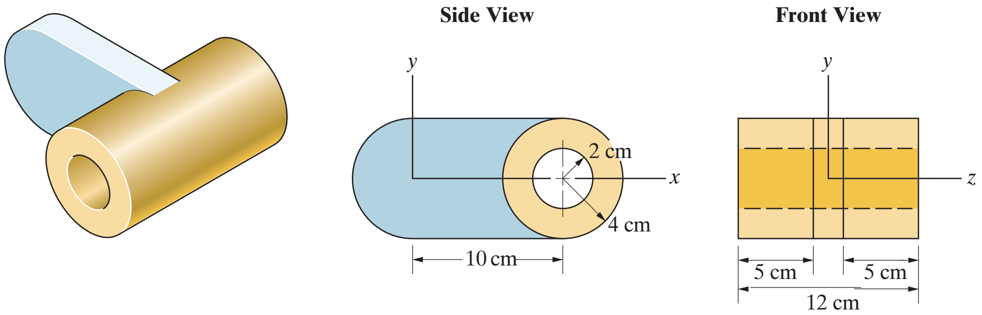
The volume of the steel plug is $V_{Fe} = \pi(0.02)^2(0.1) = 1.26 \times 10^{-4}$ m^3. The mass of the steel plug is $m_{Fe} = (7800)V_{Fe} = 0.9802$ kg. The centroid of the steel plug is $\mathbf{x}_{Fe} = 0.15$ m, $\mathbf{y}_{Fe} = \mathbf{z}_{Fe} = 0$.

The composite mass is $m = 2.38$ kg. The composite centroid is

$$\mathbf{x} = \frac{m_{Al}(0.1) + m_{Fe}(0.15)}{m} = 0.121\ \text{m}$$

$$\mathbf{y} = \mathbf{z} = 0$$

Problem 7.109 In Example 7.19, suppose that the object is redesigned so that the radius of the hole in the hollow cylinder is increased from 2 cm to 3 cm. What is the x coordinate of the center of mass of the object?



Solution: The volume of the cylinder is

$$V_{\text{cylinder}} = (12\text{ cm})\pi\,[(4\text{ cm})^2 - (3\text{ cm})^2] = 264\text{ cm}^3 = 2.64 \times 10^{-4}\,\text{m}^3$$

Its weight is

$$W_{\text{cylinder}} = (2.64 \times 10^{-4}\,\text{m}^3)\,(85\text{ kN/m}^3) = 0.02244\text{ kN}.$$

The x coordinate of the center of mass (same as center of weight) is

$$\bar{x} = \frac{\bar{x}_{\text{bar}} W_{\text{bar}} + \bar{x}_{\text{cylinder}} W_{\text{cylinder}}}{W_{\text{bar}} + W_{\text{cylinder}}}$$

$$= \frac{(1.86\text{ cm})\,(4.43 \times 10^{-4}\text{ kN}) + (10\text{ cm})\,(0.02244\text{ kN})}{0.02244\text{ kN}} = 9.84\text{ cm}.$$

$\boxed{\bar{x} = 9.84\text{ cm}.}$

Problem 7.110 A machine consists of three parts. The masses and the locations of the centers of mass of two of the parts are:

Part	Mass (kg)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)
1	2.0	100	50	−20
2	4.5	150	70	0

The mass of part 3 is 2.5 kg. The design engineer wants to position part 3 so that the center of mass location of the machine is $\bar{x} = 120$ mm, $\bar{y} = 80$ mm, and $\bar{z} = 0$. Determine the necessary position of the center of mass of part 3.

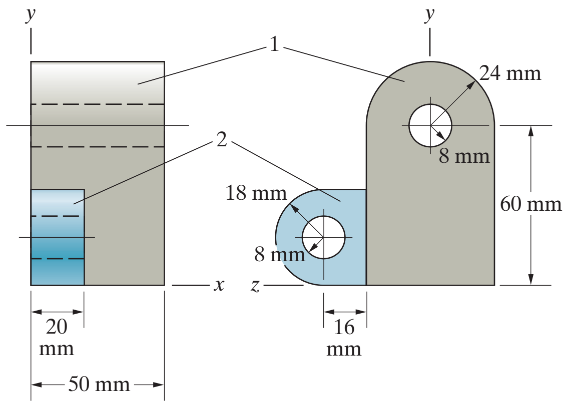
Solution: The composite mass is $m = 2.0 + 4.5 + 2.5 = 9$ kg. The location of the third part is

$$\mathbf{x}_3 = \frac{120(9) - 2(100) - 4.5(150)}{2.5} = 82\text{ mm}$$

$$\mathbf{y}_3 = \frac{80(9) - 2(50) - 4.5(70)}{2.5} = 122\text{ mm}$$

$$\mathbf{z}_3 = \frac{2(20)}{2.5} = 16\text{ mm}$$

Problem 7.111 Two views of a machine element are shown. Part 1 is aluminum alloy with density 2800 kg/m^3, and part 2 is steel with density 7800 kg/m^3. Determine the coordinates of its center of mass.



Solution: The volumes of the parts are

$$V_1 = \left[(60)(48) + \tfrac{1}{2}\pi(24)^2 - \pi(8)^2\right](50)$$

$$= 179,186 \text{ mm}^3 = 17.92 \times 10^{-5} \text{ m}^3,$$

$$V_2 = \left[(16)(36) + \tfrac{1}{2}\pi(18)^2 - \pi(8)^2\right](20) = 17,678 \text{ mm}^3$$

$$= 1.77 \times 10^{-5} \text{ m}^3,$$

so their masses are

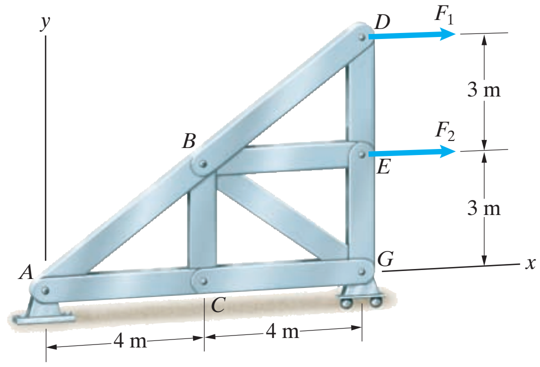
$$m_1 = S_1 V_1 = (2800)(17.92 \times 10^{-5}) = 0.502 \text{ kg},$$

$$m_2 = S_2 V_2 = (7800)(1.77 \times 10^{-5}) = 0.138 \text{ kg}.$$

The x coordinates of the centers of mass of the parts are $\mathbf{x}_1 = 25$ mm, $\mathbf{x}_2 = 10$ mm, so

$$\mathbf{x} = \frac{\mathbf{x}_1 m_1 + \mathbf{x}_2 m_2}{m_1 + m_2} = 21.8 \text{ mm}$$

Problem 7.112 The loads $F_1 = F_2 = 25$ kN. The mass of the truss is 900 kg. The members of the truss are homogeneous bars with the same uniform cross section. (a) What is the x coordinate of the center of mass of the truss? (b) Determine the reactions at A and G.



Solution:

(a) The center of mass of the truss is located at the centroid of the composite line of the axes of the members. The lengths of the diagonal members are $\sqrt{(4\text{ m})^2 + (3\text{ m})^2} = 5$ m. The lengths and x coordinates of the centroids of the axes of the members are

Member	Length	x coordinate
AB	5 m	2 m
AC	4 m	2 m
BC	3 m	4 m
BD	5 m	6 m
BE	4 m	6 m
BG	5 m	6 m
CG	4 m	6 m
DE	3 m	8 m
EG	3 m	8 m

The x coordinate of the centroid of the composite line, which is coincident with the center of mass of the truss, is

$$\bar{x} = \frac{\Sigma x_i L_i}{\Sigma L_i} = \frac{(2)\ (5+4) + (4)\ (3) + (6)\ (5+4+5+4) + (8)\ (3+3)}{5+4+3+5+4+5+4+3+3} = 5.17\text{ m}$$

$\boxed{\bar{x} = 5.17\text{ m}}$

(b) The equilibrium equations for the truss are

$\Sigma F_x : A_x + 25\text{ kN} + 25\text{ kN},$

$\Sigma F_y : A_y + G - (900)(9.81)\text{N} = 0,$

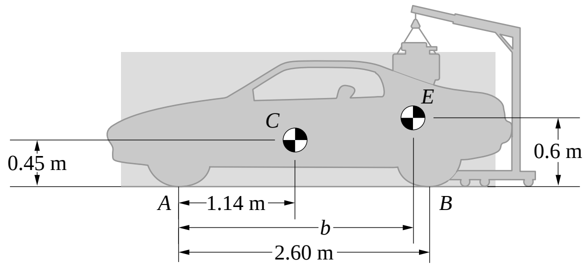
$\Sigma M_A : -(25\text{ kN})\ (3\text{ m}) - (25\text{ kN})\ (6\text{ m}) - (900)\ (9.81)\ \text{N}(5.17\text{ m}) + G(8\text{ m}) = 0.$

Solving yields $\boxed{A_{xc} = -50\text{ kN},\ A_y = -25.0\text{ kN},\ G = 33.8\text{ kN}.}$

Problem 7.113 With its engine removed, the mass of the car is 1100 kg and its center of mass is at C. The mass of the engine is 220 kg.

(a) Suppose that you want to place the center of mass E of the engine so that the center of mass of the car is midway between the front wheels A and the rear wheels B. What is the distance b?

(b) If the car is parked on a 15° slope facing up the slope, what total normal force is exerted by the road on the rear wheels B?



Solution:

(a) The composite mass is $m = m_C + m_E = 1320$ kg. The x-coordinate of the composite center of mass is given:

$$\mathbf{x} = \frac{2.6}{2} = 1.3 \text{ m},$$

from which the x-coordinate of the center of mass of the engine is

$$\mathbf{x}_E = b = \frac{(1.3 \text{ m} - 1.14\ m_C)}{m_E} = 2.1 \text{ m}.$$

The y-coordinate of the composite center of mass is

$$\mathbf{y} = \frac{0.45\ m_C + 0.6\ m_E}{m} = 0.475 \text{ m}.$$

(b) Assume that the engine has been placed in the new position, as given in Part (a). The sum of the moments about B is

$$\sum M_A = 2.6A + \mathbf{y}mg\sin(15^\circ)$$

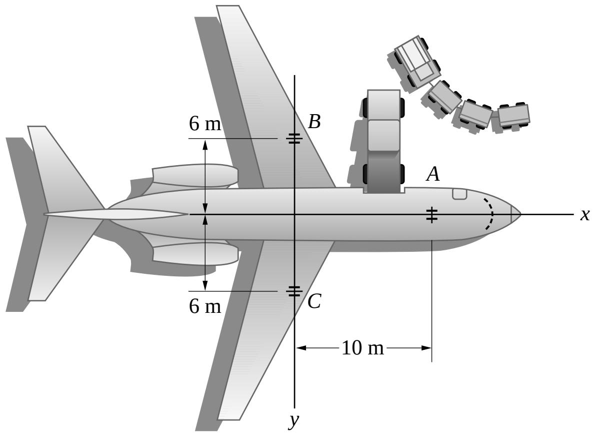
$$- (2.6 - \mathbf{x})mg\cos(15^\circ) = 0,$$

from which $A = 5641.7$ N. This is the normal force exerted by the road on A. The normal force exerted on B is obtained from;

$$\sum F_N = A - mg\cos(15^\circ) + B = 0,$$

from which $B = 6866$ N

Problem 7.114 The airplane is parked with its landing gear resting on scales. The weights measured at A, B, and C are 30 kN, 140 kN, and 146 kN, respectively. After a crate is loaded onto the plane, the weights measured at A, B, and C are 31 kN, 142 kN, and 147 kN, respectively. Determine the mass and the x and y coordinates of the center of mass of the crate.



Solution: The weight of the airplane is $W_A = 30 + 140 + 146 = 316$ kN. The center of mass of the airplane:

$$\sum M_{y\text{axis}} = 30(10) - \mathbf{x}_A W_A = 0,$$

from which $\mathbf{x}_A = 0.949$ m.

$$\sum M_{x\text{axis}} = (140 - 146)(6) + \mathbf{y}_A W_A = 0,$$

from which $\mathbf{y}_A = 0.114$ m. The weight of the loaded plane:

$$W = 31 + 142 + 147 = 320 \text{ kN}.$$

The center of mass of the loaded plane:

$$\sum M_{y\text{axis}} = (31)10 - \mathbf{x}W = 0,$$

from which $\mathbf{x} = 0.969$ m.

$$\sum M_{x\text{axis}} = (142 - 147)(6) + \mathbf{y}W = 0,$$

from which $\mathbf{y} = 0.0938$ m. The weight of the crate is $W_c = W - W_A = 4$ kN. The center of mass of the crate:

$$\mathbf{x}_c = \frac{W\mathbf{x} - W_A\mathbf{x}_A}{W_c} = 2.5 \text{ m},$$

$$\mathbf{y}_c = \frac{W\mathbf{y} - W_A\mathbf{y}_A}{W_c} = -1.5 \text{ m}.$$

The mass of the crate:

$$m_c = \frac{W_c \times 10^3}{9.81} = 407.75 \text{ kg}$$

Problem 7.115 A suitcase with a mass of 90 kg is placed in the trunk of the car described in Example 7.20. The position of the center of mass of the suitcase is $\bar{x}_s = -0.533$ m, $\bar{y}_s = 0.762$ m, and $\bar{z}_s = -0.305$ m. If the suitcase is regarded as part of the car, what is the new position of the car's center of mass?

Solution: In Example 7.20, the following results were obtained for the car without the suitcase

$$W_c = 17303 \text{ N}$$

$$\mathbf{x}_c = 1.651 \text{ m}$$

$$\mathbf{y}_c = 0.584 \text{ m}$$

$$\mathbf{z}_c = 0.769 \text{ m}$$

For the suitcase

$$W_s = (90)\text{ g}, \quad \mathbf{x}_s = -0.533 \text{ m},$$

$$\mathbf{y} = 0.762 \text{ m}, \quad \mathbf{z} = -0.305 \text{ m}.$$

The new center of mass is at

$$\mathbf{x}_N = \frac{\mathbf{x}_c W_c + \mathbf{x}_s W_s}{(W_c + W_s)}$$

with similar eqns for $\mathbf{y}_N$ and $\mathbf{z}_N$

Solving, we get

$$\mathbf{x}_N = 1.545 \text{ m}, \ \mathbf{y}_N = 0.593 \text{ m}, \ \mathbf{z}_N = 0.717 \text{ m}$$

Problem 7.116 A group of engineering students constructs a miniature device of the kind described in Example 7.20 and uses it to determine the center of mass of a miniature vehicle. The data they obtain are shown in the following table:

Wheelbase = 0.9 m		
Track = 0.75 m	**Measured Loads (N)**	
	$\alpha = 0$	$\alpha = 10^\circ$
Left front wheel, N_{LF}	157	144
Right front wheel, N_{RF}	162	149
Left rear wheel, N_{LR}	122	153
Right rear wheel, N_{RR}	131	135

Determine the center of mass of the vehicle. Use the same coordinate system as in Example 7.20.

Solution: The weight of the go-cart: $W = 157 + 162 + 122 + 131 = 572$ N. The sum of the moments about the z axis

$$\sum M_{z\text{axis}} = (Wheelbase)(N_{LF} + N_{RF}) - \mathbf{x}W = 0,$$

from which

$$\mathbf{x} = \frac{0.9(157 + 162)}{W} = 0.502 \text{ m}.$$

The sum of the moments about the x axis:

$$\sum M_{x\text{axis}} = \mathbf{z}W - (Track)(N_{RF} + N_{RR}) = 0,$$

from which

$$\mathbf{z} = \frac{(0.75)(162 + 131)}{W} = 0.384 \text{ m}.$$

With the go-cart in the tilted position, the sum of the moments about the z axis

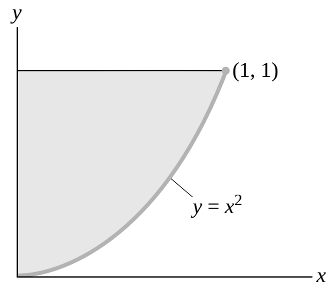
$$\sum M_{z\text{axis}} = (Wheelbase)(N_{LF} + N_{RF}) + \mathbf{y}W\sin(10^\circ) - \mathbf{x}W\cos(10^\circ) = 0,$$

from which

$$\mathbf{y} = \frac{\mathbf{x}W\cos(10^\circ) - (0.9)(144 + 149)}{W\sin(10^\circ)}$$

$$= 0.192 \text{ m}.$$

Problem 7.117 Determine the centroid of the area by letting dA be a vertical strip of width dx.



Solution: *The area:* The length of the vertical strip is $(1 - y)$, so that the elemental area is $dA = (1 - y)\,dx = (1 - x^2)\,dx$. The area:

$$\int_A dA = \int_0^1 (1 - x^2)\,dx = \left[x - \frac{x^3}{3}\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}.$$

The x-coordinate:

$$\mathbf{x}A = \int_A x\,dA = \int_0^1 x(1 - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{4} : \ \mathbf{x} = \frac{3}{8}$$

The y-coordinate: The y-coordinate of the centroid of each element of area is located at the midpoint of the vertical dimension of the area element.

$$\mathbf{y} = y + \tfrac{1}{2}(1 - x^2).$$

Thus

$$\int_A \mathbf{y}\,dA = \int_0^1 \left(x^2 + \left(\frac{1}{2}\right)(1 - x^2)\right)(1 - x^2)\,dx$$

$$= \left(\frac{1}{2}\right)\left[x - \frac{x^5}{5}\right]_0^1 = \frac{2}{5}.$$

$$\mathbf{y} = \frac{3}{5}$$

Problem 7.118 Determine the centroid of the area in Problem 7.117 by letting dA be a horizontal strip of height dy.

Solution: *The area:* The length of the horizontal strip is x, hence the element of area is

$$dA = x\,dy = y^{1/2}\,dy.$$

Thus

$$A = \int_0^1 y^{1/2}\,dy = \left[\frac{2y^{3/2}}{3}\right]_0^1 = \frac{2}{3}$$

Check:

The x-coordinate: The x-coordinate of the centroid of each element of area is $\mathbf{x} = \frac{1}{2}x = \frac{1}{2}y^{1/2}$. Thus

$$\int_A \left(\frac{1}{2}\right) y^{1/2}\,dA = \left(\frac{1}{2}\right)\int_0^1 y\,dy = \left(\frac{1}{2}\right)\left[\frac{y^2}{2}\right]_0^1 = \frac{1}{4}.$$

Divide by the area: $\mathbf{x} = \dfrac{3}{8}$

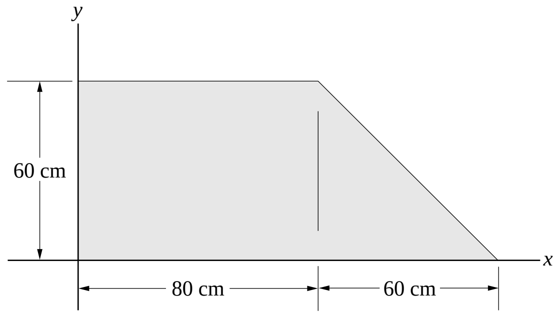
The y-coordinate:

$$\mathbf{y}A = \int_A y\,dA = \int_0^1 y\left(y^{1/2}\,dy\right)$$

$$= \int_0^1 y^{3/2}\,dy = \left[\frac{2y^{5/2}}{5}\right]_0^1 = \frac{2}{5}.$$

Divide by the area: $\mathbf{y} = \dfrac{3}{5}$

Problem 7.119 Determine the centroid of the area.



Solution: The strategy is to develop useful general results for the triangle and the rectangle.

The rectangle: The area of the rectangle of height h and width w is

$$A = \int_0^w h\,dx = hw = 4800 \text{ cm}^2.$$

The x-coordinate:

$$\int_0^w hx\,dx = h\left[\frac{x^2}{2}\right]_0^w = \left(\frac{1}{2}\right) hw^2.$$

Divide by the area: $\mathbf{x} = \frac{w}{2} = 40$ cm

The y-coordinate:

$$\left(\frac{1}{2}\right)\int_0^w h^2\,dx = \left(\frac{1}{2}\right) h^2 w.$$

Divide by the area: $\mathbf{y} = \left(\frac{1}{2}\right) h = 30$ cm

The triangle: The area of the triangle of altitude a and base b is (assuming that the two sides a and b meet at the origin)

$$A = \int_0^b y(x)\,dx = \int_0^b \left(-\frac{a}{b}x + a\right)\,dx = \left[-\frac{ax^2}{2b} - ax\right]_0^b$$

$$= \left[-\frac{ab}{2} + ab\right] = \frac{ab}{2} = 1800 \text{ cm}^2$$

Check: This is the familiar result. *check.*

The x-coordinate:

$$\int_0^b \left(-\frac{a}{b}x + a\right) x\,dx = \left[-\frac{ax^3}{3b} + \frac{ax^2}{2}\right]_0^b = \frac{ab^2}{6}.$$

Divide by the area: $\mathbf{x} = \frac{b}{3} = 20$ cm

The y-coordinate:

$$\int_A y\,dA = \left(\frac{1}{2}\right)\int_0^b \left(-\frac{a}{b}x + a\right)^2\,dx$$

$$= -\frac{b}{6a}\left[\left(-\frac{a}{b}x + a\right)^3\right]_0^b = \frac{ba^2}{6}.$$

Divide by the area: $\mathbf{y} = \frac{a}{3} 20$ cm. *The composite:*

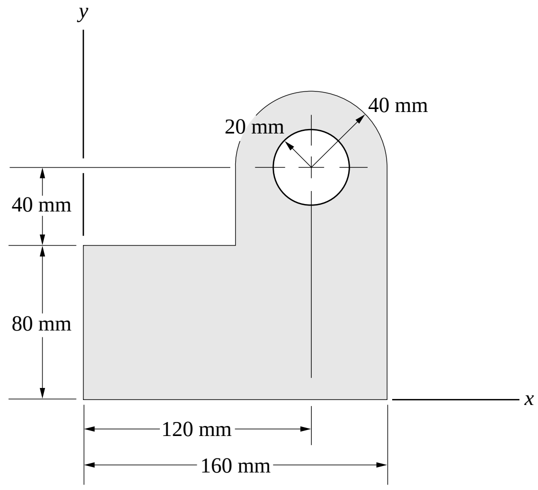
$$\mathbf{x} = \frac{\mathbf{x}_R A_R + \mathbf{x}_T A_T}{A_R + A_T} = \frac{40(4800) + 100(1800)}{4800 + 1800}$$

$$\boxed{= 56.36 \text{ cm}}$$

$$\mathbf{y} = \frac{(30)(4800) + (20)(1800)}{4800 + 1800}$$

$$\boxed{= 27.27 \text{ cm}}$$

Problem 7.120 Determine the centroid of the area.



Solution: Divide the object into five areas:

(1) The rectangle 80 mm by 80 mm,
(2) The rectangle 120 mm by 80 mm,
(3) the semicircle of radius 40 mm,
(4) The circle of 20 mm radius, and
(5) the composite object. The areas and centroids:

(1) $A_1 = 6400$ mm^2,
$\mathbf{x}_1 = 40$ mm, $\mathbf{y}_1 = 40$ mm,

(2) $A_2 = 9600$ mm^2,
$\mathbf{x}_2 = 120$ mm, $\mathbf{y}_2 = 60$ mm,

(3) $A_3 = 2513.3$ mm^2,
$\mathbf{x}_3 = 120$ mm, $\mathbf{y}_3 = 136.98$ mm,

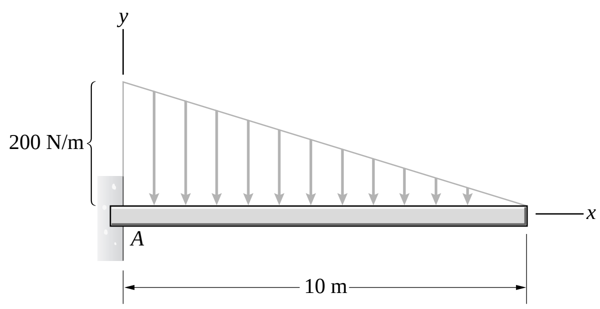
(4) $A_4 = 1256.6$ mm^2,
$\mathbf{x}_4 = 120$ mm, $\mathbf{y}_4 = 120$ mm.

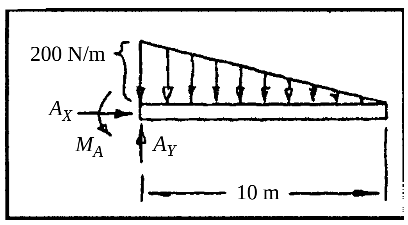
(5) The composite area: $A = A_1 + A_2 + A_3 - A_4 = 17256.6$ mm^2.
The composite centroid:

$$\boxed{\mathbf{x} = \frac{A_1\mathbf{x}_1 + A_2\mathbf{x}_2 + A_3\mathbf{x}_3 - A_4\mathbf{x}_4}{A} = 90.3 \text{ mm}}.$$

$$\boxed{\mathbf{y} = \frac{A_1\mathbf{y}_1 + A_2\mathbf{y}_2 + A_3\mathbf{y}_3 - A_4\mathbf{y}_4}{A} = 59.4 \text{ mm}}$$

Problem 7.121 The cantilever beam is subjected to a triangular distributed load. What are the reactions at A?





Solution: The load distribution is a straight line with intercept $w = 200$ N/m at $x = 0$, and slope $-\left(\frac{200}{10}\right) = -20$ N/m^2. The sum of the moments is

$$\sum M = M_A - \int_0^{10} (-20x + 200)x\, dx = 0,$$

from which

$$M_A = \left[-\frac{20}{3}x^3 + 100x^2\right]_0^{10} = 3333.3 \text{ N-m.}$$

The sum of the forces:

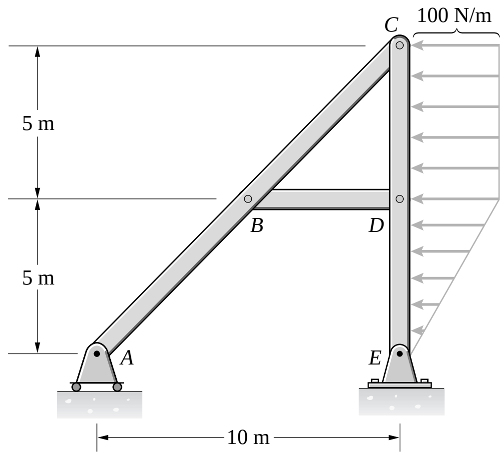
$$\sum F_y = A_y - \int_0^{10} (-20x + 200)\, dx = 0,$$

from which

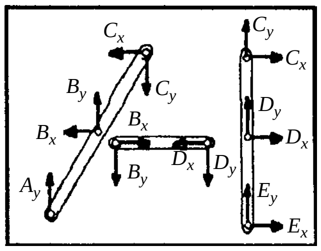
$$A_y = \left[-10x^2 + 200x\right]_0^{10} = 1000 \text{ N},$$

and $\sum F_x = A_x = 0$

Problem 7.122 What is the axial load in member *BD* of the frame?



Solution: The distributed load is two straight lines: Over the interval $0 \le y \le 5$ the intercept is $w = 0$ at $y = 0$ and the slope is $+\frac{100}{5} = 20$.



Over the interval $5 \le y \le 10$, the load is a constant $w = 100$ N/m. The moment about the origin E due to the load is

$$M_E = \int_0^5 (20y)y\,dy + \int_5^{10} 100y\,dy,$$

from which

$$M_E = \left[\frac{20}{3}y^3\right]_0^5 + \left[\frac{100}{2}y^2\right]_5^{10} = 4583.33 \text{ N-m}.$$

Check: The area of the triangle is

$$F_1 = (\tfrac{1}{2})(5)(100) = 250 \text{ N}.$$

The area of the rectangle: $F_2 = 500$ N. The centroid distance for the triangle is

$$d_1 = (\tfrac{2}{3})5 = 3.333 \text{ m}.$$

The centroid distance of the rectangle is $d_2 = 7.5$ m. The moment about E is

$$M_E = d_1F_1 + d_2F_2 = 4583.33 \text{ Nm } \textit{check}.$$

The Complete Structure: The sum of the moments about E is

$$\sum M = -10A_R + M_E = 0,$$

where A_R is the reaction at A, from which $A_R = 458.33$ N.

The element ABC: Element *BD* is a two force member, hence $B_y = 0$. The sum of the moments about C:

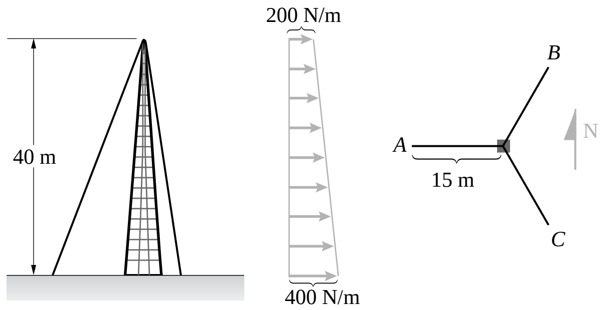
$$\sum M_C = -5B_x - 10A_y = 0,$$

where A_y is equal and opposite to the reaction of the support, from which

$$B_x = -2A_y = 2A_R = 916.67 \text{ N}.$$

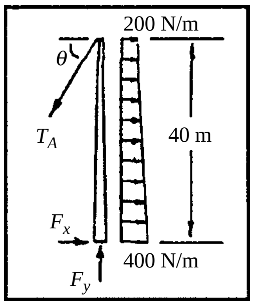
Since the reaction in element *BD* is equal and opposite, $B_x = -916.67$ N, *which is a tension in BD.*

Problem 7.123 An engineer estimates that the maximum wind load on the 40-m tower in Fig. a is described by the distributed load in Fig. b. The tower is supported by three cables A, B, and C from the top of the tower to equally spaced points 15 m from the bottom of the tower (Fig. c). If the wind blows from the west and cables B and C are slack, what is the tension in cable A? (Model the base of the tower as a ball and socket support.)



(a) (b) (c)

Solution: The load distribution is a straight line with the intercept $w = 400$ N/m, and slope -5. The moment about the base of the tower due to the wind load is



$$M_W = \int_0^{40} (-5y + 400) y \, dy,$$

$$M_W = \left[-\frac{5}{3}y^3 + 200y^2\right]_0^{40} = 213.33 \text{ kN-m},$$

clockwise about the base, looking North. The angle formed by the cable with the horizontal at the top of the tower is

$$\theta = 90^\circ - \tan^{-1}\left(\frac{15}{40}\right) = 69.44^\circ.$$

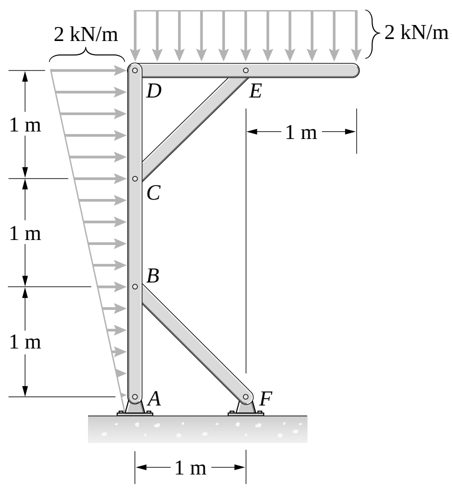
The sum of the moments about the base of the tower is

$$\sum M = -M_W + 40T_A \cos\theta = 0,$$

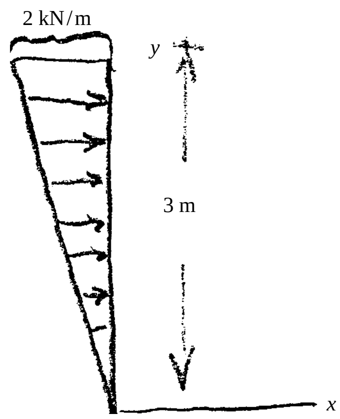
from which

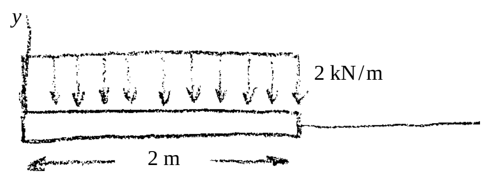
$$T_A = \left(\frac{1}{40\cos\theta}\right) M_W = 15.19 \text{ kN}$$

Problem 7.124 Determine the reactions on member $ABCD$ at A and D.



Solution: First, replace the distributed forces with equivalent concentrated forces, then solve for the loads. Note that BF and CE are two force members.





Distributed Load on $ABCD$, F_1

By area analogy, concentrated load is applied at $y = \pm m$. The load is $\frac{1}{2}$ (2)(3) kN

$F_1 = 3$ kN

By the area analogy,

$F_2 = 4$ kN applied at $x = 1$ m

Assume F_{CE} and F_{BF} are tensions

For $ABCD$:

$$\sum F_x: \quad A_x + F_{BF}\cos 45^\circ + F_{CE}\cos 45^\circ + D_x + 3 \text{ kN} = 0$$

$$\sum F_y: \quad A_y + D_y - F_{BF}\sin 45^\circ + F_{CE}\sin 45^\circ = 0$$

$$\sum M_A: \quad -1(F_{BF}\cos 45^\circ) - 2(F_{CE}\cos 45^\circ) - 2(3) - 3D_x = 0$$

For DE:

$$\sum F_x: \quad -D_x - F_{CE}\cos 45^\circ = 0$$

$$\sum F_y: \quad -D_y - F_{CE}\sin 45^\circ - 4 = 0$$

$$\sum M_E: \quad (1)D_y = 0$$

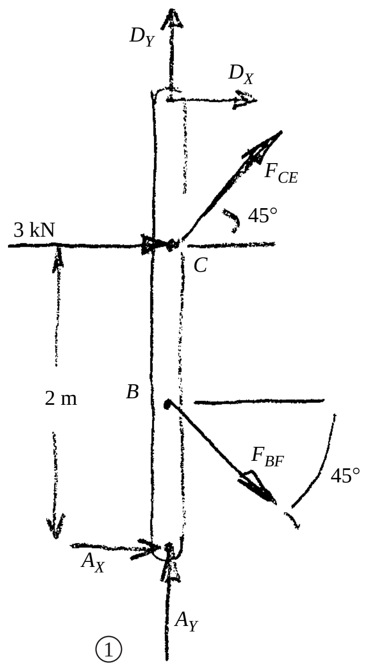
7.124 (*Continued*)

Solving, we get

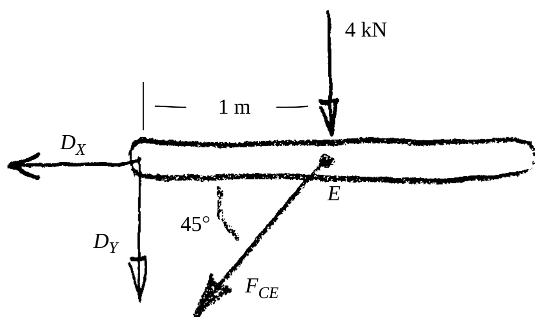
$$\boxed{\begin{aligned} A_x &= 7 \text{ kN} \\ A_y &= -6 \text{ kN} \\ D_x &= 4 \text{ kN} \\ D_y &= 0 \end{aligned}}$$

also $F_{BF} = -14.14$ kN(c)

$F_{CE} = -5.66$ kN(c)

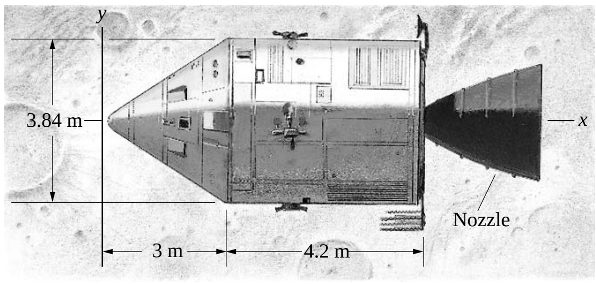


1



2

Problem 7.125 Estimate the centroid of the volume of the *Apollo* lunar return configuration (not including its rocket nozzle) by treating it as a cone and a cylinder.



Solution: The volume of the cone is

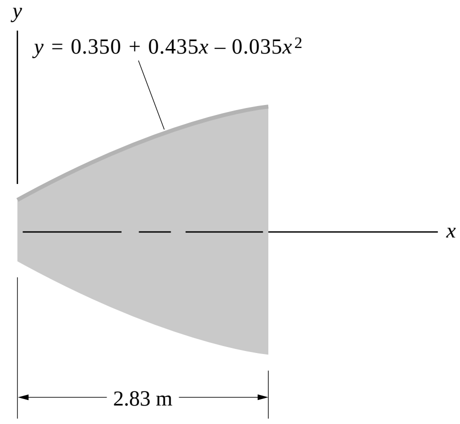
$$V_1 = \frac{\pi R^2 h}{3} = 11.581\ \text{m}^3.$$

The x-coordinate of the centroid from the nose of the cone is $\mathbf{x}_1 = \frac{3h}{4} = 2.25$ m. The volume of the cylinder is $V_2 = \pi R^2 L = 48.641\ \text{m}^3$. The x-coordinate of the centroid from the nose of the cone is $\mathbf{x}_2 = h + \frac{L}{2} = 5.1$ m. The composite volume is $V = V_1 + V_2 = 60.222\ \text{m}^3$. The x-coordinate of the composite centroid is

$$\boxed{\mathbf{x} = \frac{V_1 x_1 + V_2 x_2}{V} = 4.552\ \text{m}}.$$

The y- and z-coordinates are zero, from symmetry.

Problem 7.126 The shape of the rocket nozzle of the *Apollo* lunar return configuration is approximated by revolving the curve shown around the x axis. In terms of the coordinate system shown, determine the centroid of the volume of the nozzle.



Solution:

$$\mathbf{x} = \frac{\int x\,dV}{\int dV}.$$

Let dV be a disk of radius y and thickness dx

Thus, $dV = \pi y^2\,dx$, where

$y^2 = (0.350 + 0.435x - 0.035x^2)^2$

$y^2 = (a + bx + cx^2)^2$

$y^2 = a^2 + 2abx + (2ac + b^2)x^2 + 2bcx^3 + c^2x^4$

$a = 0.350$

$b = 0.435$

$c = -0.035$

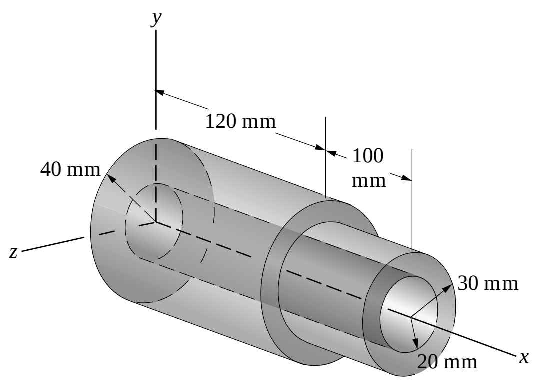
$$\mathbf{x} = \frac{\int_0^{2.83} (a^2x + 2abx^2 + (2ac + b^2)x^3 + 2bcx^4 + c^2x^5)\,dx}{\int_0^{2.83} (a^2 + 2abx + (2ac + b^2)x^2 + 2bcx^3 + c^2x^4)\,dx}$$

$$\mathbf{x} = \frac{\left[a^2\left(\frac{x^2}{2}\right) + 2ab\left(\frac{x^3}{3}\right) + (2ac + b^2)\left(\frac{x^4}{4}\right) + 2bc\left(\frac{x^5}{5}\right) + c^2\left(\frac{x^6}{6}\right) \right]_0^{2.83}}{\left[a^2x + 2ab\left(\frac{x^2}{2}\right) + (2ac + b^2)\left(\frac{x^3}{3}\right) + 2bc\left(\frac{x^4}{4}\right) + c^2\left(\frac{x^5}{5}\right) \right]_0^{2.83}}$$

Evaluating,

$$\mathbf{x} = \frac{4.43}{3.37} = 1.87 \text{ m}$$

Problem 7.127 Determine the coordinates of the centroid of the volume.

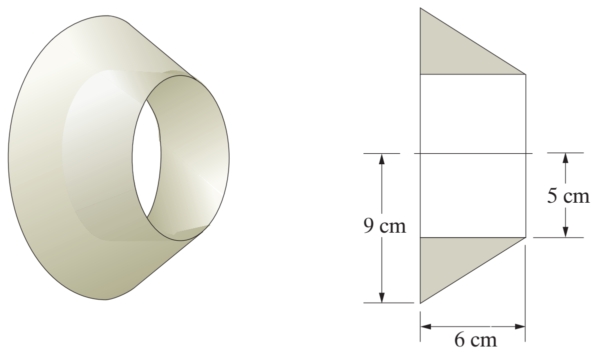


Solution: From symmetry $\overline{y} = \overline{z} = 0$

$$\overline{x} = \frac{\pi(0.04^2 - 0.03^2)(0.12)(0.06) + \pi(0.03^2 - 0.02^2)(0.22)(0.11)}{\pi(0.04^2 - 0.03^2)(0.12) + \pi(0.03^2 - 0.02^2)(0.22)}$$

$\overline{x} = 0.0884$ m $= 88.4$ mm, $\overline{y} = \overline{z} = 0$

Problem 7.128 Determine the surface area of the volume of revolution in Problem 7.127.

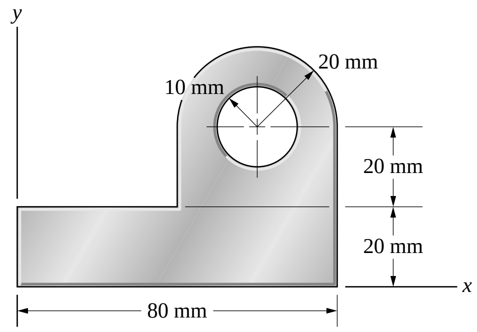


Solution: *The outer surface:* The length of the line is $L_1 = \sqrt{4^2 + 6^2} = 7.2111$ cm. The y-coordinate of the centroid is $\mathbf{y} = 5 + 2 = 7$ cm. The surface area is $A_1 = 2\pi L_1 \mathbf{y}_1 = 317.16$ cm^2.

The side surface: The length of the line is $L_2 = 4$ cm. The y-coordinate of the centroid is $\mathbf{y}_2 = 5 + 2 = 7$ cm. The surface area is $A_2 = 2\pi L_2 \mathbf{y}_2 = 87.96$ cm^2.

The inner surface: The length of the line is $L_3 = 6$ cm. The y-coordinate is $\mathbf{y}_3 = 5$ cm. The surface area is $A_3 = 2\pi L_3 \mathbf{y}_3 = 188.5$ cm^2. The total surface: $\boxed{A = A_1 + A_2 + A_3 = 681.6 \text{ cm}^2}$

Problem 7.129 Determine the y coordinate of the center of mass of the homogeneous steel plate.



Solution: Divide the object into five areas: (1) The lower rectangle 20 by 80 mm, (2) an upper rectangle, 20 by 40 mm, (3) the semicircle of radius 20 mm, (4) the circle of radius 10 mm, and (5) the composite part. The areas and the centroids are tabulated. The last row is the composite and the centroid of the composite. The composite area is

$$A = \sum_{1}^{3} A_i - A_4.$$

The centroid:

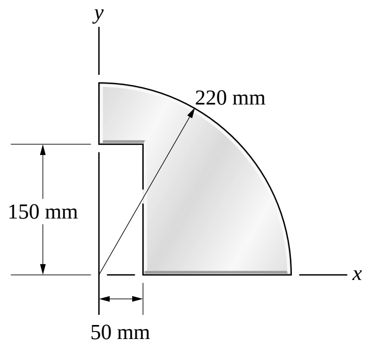
$$\mathbf{x} = \frac{\sum_{1}^{3} A_i \mathbf{x}_i - A_4 \mathbf{x}_4}{A},$$

and $\mathbf{y} = \dfrac{\sum_{1}^{3} A_i \mathbf{y}_i - A_4 \mathbf{y}_4}{A}.$

The following relationships were used for the centroids: For a rectangle: the centroid is at half the side and half the base. For a semicircle, the centroid is on the centerline and at $\frac{4R}{3\pi}$ from the base. For a circle, the centroid is at the center.

Area	A, sq mm	x, mm	y, mm
A1	1600	40	10
A2	800	60	30
A3	628.3	60	48.5
A4	314.2	60	40
Composite	2714	48.2	21.3

Problem 7.130 Determine the x coordinate of the center of mass of the homogeneous steel plate.



Solution: *The quarter circle:* The equation of the circle is $x^2 + y^2 = R^2$. Take the elemental area to be a vertical strip of height $y = \sqrt{R^2 - x^2}$ and width dx, hence the element of area is $dA = \sqrt{R^2 - x^2}\,dx$, and the area is

$$A = \int_0^R \sqrt{R^2 - x^2}\,dx = \left[\frac{x\sqrt{R^2 - x^2}}{2} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right]_0^R = \frac{\pi R^2}{4}$$

The x-coordinate:

$$\mathbf{x}_C A = \int_A x\,dA = \int_0^R x\sqrt{R^2 - x^2}\,dx = \left[-\frac{(R^2 - x^2)^{3/2}}{3}\right]_0^R = \frac{R^3}{3}$$

$$\mathbf{x}_C = \frac{4R}{3\pi}$$

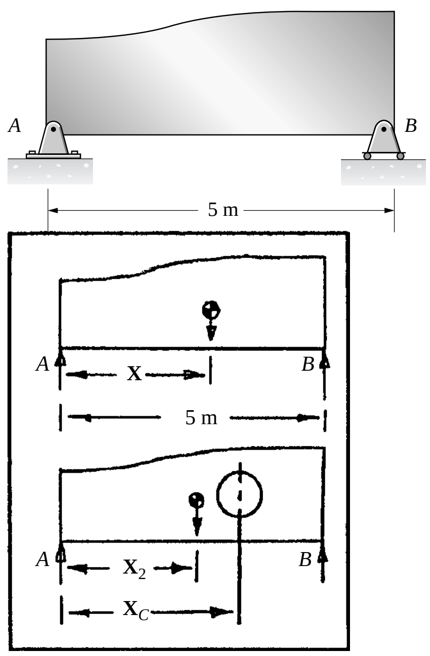
The rectangle: The area is $A = 50(150) = 7500$ mm^2. The x-coordinate of the centroid is $\mathbf{x}_R = 25$ mm.
The composite: The area of the quarter circle is

$$A_C = \frac{\pi(220)^2}{4} = 3.8013 \times 10^4 \text{ mm.}$$

The area of the rectangle is $A_R = 50(150) = 0.75 \times 10^4$ mm^2. The composite area is $A = A_C - A_R = 3.0513 \times 10^4$ mm^2. The centroid:

$$\boxed{\mathbf{x} = \frac{A_C\mathbf{x}_C - A_R\mathbf{x}_R}{A} = 110 \text{ mm}}$$

Problem 7.131 The area of the homogeneous plate is 10 m^2. The vertical reactions on the plate at A and B are 80 N and 84 N, respectively. Suppose that you want to equalize the reactions at A and B by drilling a 1-m-diameter hole in the plate. What horizontal distance from A should the center of the hole be? What are the resulting reactions at A and B?



Solution: The weight of the plate is $W = 80 + 84 = 164$ N. From the sum of moments about A, the centroid is

$$\mathbf{x} = \frac{84(5)}{W} = 2.56 \text{ m.}$$

The weight density is

$$w = \frac{W}{10} = 16.4 \text{ N/m}^2.$$

The weight of the cutout is $W_C = \pi\,(0.5^2)w = 12.88$ N. The new weight of the plate is $W_2 = W - W_C = 151.12$ N. The new centroid must be at

$$\mathbf{x}_2 = \frac{5}{2} = 2.5 \text{ m for the reactions to be equal.}$$

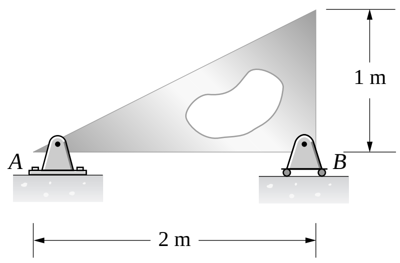
Therefore the x-coordinate of the center of the circle will be

$$\mathbf{x}_C = \frac{W\mathbf{x} - W_2\mathbf{x}_2}{W_C} = 3.26 \text{ m.}$$

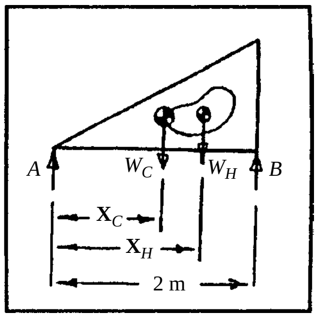
The reactions at A and B will be

$$A = B = \frac{W_2}{2} = 75.56 \text{ N}$$

Problem 7.132 The plate is of uniform thickness and is made of homogeneous material whose mass per unit area of the plate is 2 kg/m^2. The vertical reactions at A and B are 6 N and 10 N, respectively. What is the x coordinate of the centroid of the hole?



Solution: Choose an origin at A. The basic relation is $W_C\mathbf{x}_C = W\mathbf{x} - W_H\mathbf{x}_H$, where W_C is the weight of the composite plate (the one with the hole), W is the weight of the plate without the hole, W_H is the weight of the material removed from the hole, and $\mathbf{x}_C$, $\mathbf{x}$, and $\mathbf{x}_H$ are the x-coordinates of the centroids of the composite plate, the plate without the hole, and the hole, respectively.



The composite weight:

$$\sum F_Y = A + B - W_C = 0,$$

from which $W_C = 16$ N. The x-coordinate of the centroid:

$$\sum M_A = -W_C\mathbf{x}_C + 2B = 0,$$

from which $\mathbf{x}_C = 1.25$ m. The weight of the plate without the hole and the x-coordinate of the centroid:

$$W = \rho A g = \left(\tfrac{1}{2}\right)(2)(1)(2)(9.81) = 19.62 \text{ N},$$

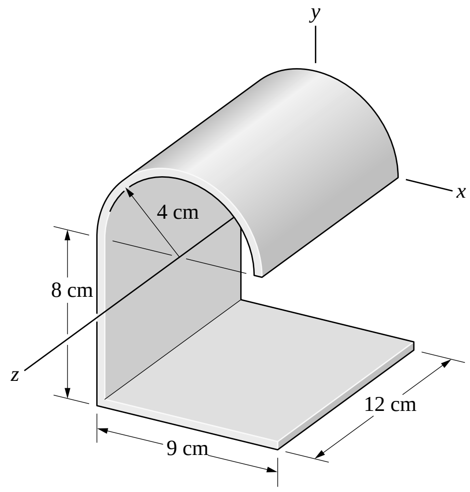
and $\mathbf{x} = \left(\tfrac{2}{3}\right)2 = 1.3333$ m.

The weight of the material removed from the hole:

$$W_H = W - W_C = 3.62 \text{ N}.$$

Solve: $\mathbf{x}_H = \dfrac{W\mathbf{x} - W_C\mathbf{x}_C}{W_H} = 1.702$ m

Problem 7.133 Determine the center of mass of the homogeneous sheet of metal.



Solution: Divide the object into four parts: (1) The lower plate, (2) the left hand plate, (3) the semicircular plate, and (4) the composite plate. The areas and centroids are found by inspection:

(1) Area: $A_1 = 9(12) = 108\ \text{cm}^2$,
$\mathbf{x}_1 = 0.5\ \text{cm}$, $\mathbf{y}_1 = -8\ \text{cm}$, $\mathbf{z}_1 = 6\ \text{cm}$.

(2) $A_2 = 8(12) = 96\ \text{cm}^2$,
$\mathbf{x}_2 = -4\ \text{cm}$,
$\mathbf{y}_2 = -4\ \text{cm}$, $\mathbf{z}_2 = 6\ \text{cm}$.

(3) $A_3 = \pi 4(12) = 150.8\ \text{cm}^2$,
$\mathbf{x}_3 = 0$, $\mathbf{y} = \dfrac{2(4)}{\pi} = 2.546\ \text{cm}$, $z = 6\ \text{cm}$.

The composite area is

$$A = \sum_1^3 A_i = 354.796\ \text{cm}^2.$$

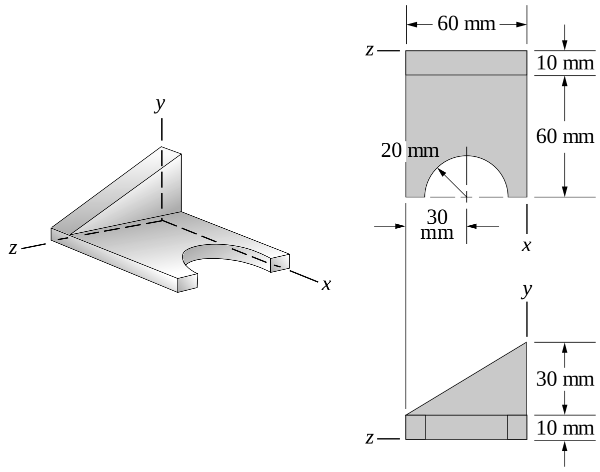
The centroid for the composite:

$$\mathbf{x} = \frac{\sum_1^3 A_i \mathbf{x}_i}{A} = -0.930\ \text{cm}$$

$$\mathbf{y} = \frac{\sum_1^3 A_i \mathbf{y}_i}{A} = -2.435\ \text{cm}$$

$$\mathbf{z} = \frac{\sum_1^3 A_i \mathbf{z}_i}{A} = 6\ \text{cm}$$

Problem 7.134 Determine the center of mass of the homogeneous object.



Solution: Divide the object into three parts and the composite: (1) A triangular solid 30 mm altitude, 60 mm base, and 10 mm thick. (2) A rectangle 60 by 70 mm by 10 mm. (3) A semicircle with radius 20 mm and 10 mm thick.

The volumes and their centroids are determined by inspection:

(1) $V_1 = \left(\frac{1}{2}\right)(30)(60)(10) = 9000\ \text{mm}^3,$

$\mathbf{x}_1 = 5\ \text{mm},$

$\mathbf{y}_1 = 10 + \frac{30}{3} = 20\ \text{mm},$

$\mathbf{z}_1 = \frac{60}{3} = 20\ \text{mm}$

(2) $V_2 = 60(70)(10) = 42000\ \text{mm}^3,$

$\mathbf{x}_2 = 35\ \text{mm},$

$\mathbf{y}_2 = 5\ \text{mm},$

$\mathbf{z}_2 = 30\ \text{mm}$

(3) $V_3 = \frac{\pi 20^2}{2}(10) = 6283.2\ \text{mm}^3,$

$\mathbf{x}_3 = 70 - \frac{4(20)}{3\pi} = 61.51\ \text{mm},$

$\mathbf{y}_3 = 5\ \text{mm},$

$\mathbf{z}_3 = 30\ \text{mm}.$

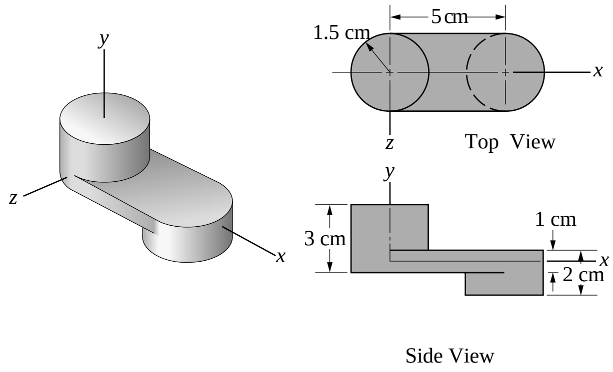
The composite volume is $V = V_1 + V_2 - V_3 = 44716.8\ \text{mm}^3$. The centroid is

$$\mathbf{x} = \frac{V_1\mathbf{x}_1 + V_2\mathbf{x}_2 - V_3\mathbf{x}_3}{V} = 25.237\ \text{mm}$$

$$\mathbf{y} = \frac{V_1\mathbf{y}_1 + V_2\mathbf{y}_2 - V_3\mathbf{y}_3}{V} = 8.019\ \text{mm}$$

$$\mathbf{z} = \frac{V_1\mathbf{z}_1 + V_2\mathbf{z}_2 - V_3\mathbf{z}_3}{V} = 27.99\ \text{mm}$$

Problem 7.135 Determine the center of mass of the homogeneous object.



Solution: Divide the object into five parts plus the composite. (1) A solid cylinder with 1.5 cm radius, 3 cm long. (2) A rectangle 3 by 5 by 1 cm (3) A solid cylinder with radius 1.5 cm, 2 cm long. (4) A semicircle with radius 1.5 cm, 1 cm thick, (5) a semicircle with radius 1.5 cm, 1 cm thick. The volumes and centroids are determined by inspection. These are tabulated:

Part No	Vol, cu cm	**x**, cm	**y**, cm	**z**, cm
V1	21.205	0	1	0
V2	15	2.5	0	0
V3	14.137	5	−0.5	0
V4	3.534	0.6366	0	0
V5	3.534	4.363	0	0
Composite	43.27	2.09	0.3267	0

The composite is

$$V = \sum_{1}^{3} V_i - \sum_{4}^{5} V_i.$$

The centroid:

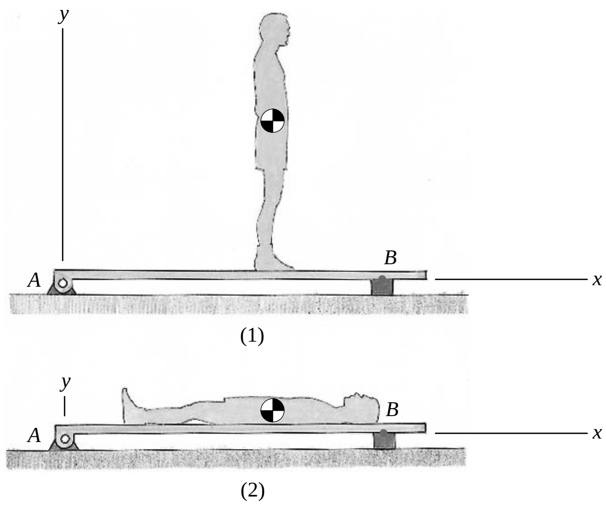
$$\mathbf{x} = \frac{\sum_{1}^{3} V_i \mathbf{x}_i - \sum_{4}^{5} V_i \mathbf{x}_i}{V},$$

with a corresponding expression for **y**. The z-coordinate is zero because of symmetry.

Problem 7.136 The arrangement shown can be used to determine the location of the center of mass of a person. A horizontal board has a pin support at A and rests on a scale that measures weight at B. The distance from A to B is 2.3 m. When the person is not on the board, the scale at B measures 90 N.

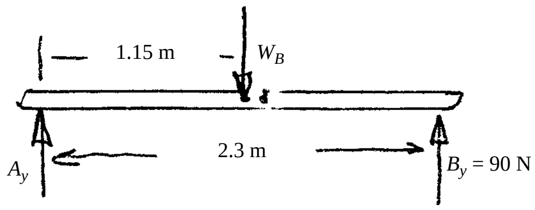
(a) When a 63-kg person is in position (1), the scale at B measures 496 N. What is the x coordinate of the person's center of mass?

(b) When the same person is in position (2), the scale measures 523 N. What is the x coordinate of his center of mass?



Solution:

$W = mg = 63\text{ g} = 618\text{ N}$

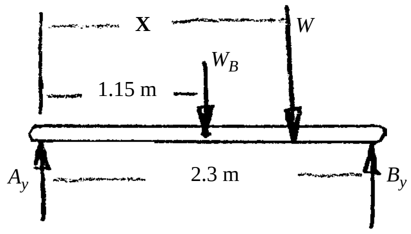


(a) Unloaded Beam (assume uniform beam)

$$\sum F_y: \quad A_y + B_y - W_B = 0$$

$$\sum M_A: \quad (-1.15)W_B + 2.3B_y = 0$$

Solving, $A_y = 90$ N, $W_B = 180$ N



(b)

$$\sum F_y: \quad A_y + B_y - W_B - W = 0$$

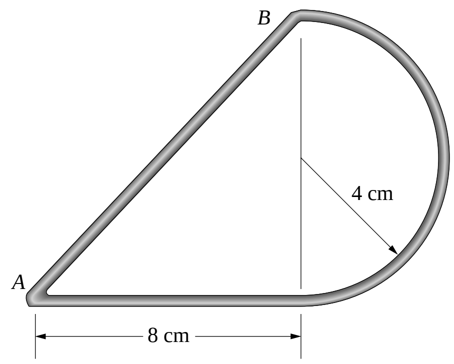
$$\sum M_A: \quad 2.3B_y - 1.15W_B - \mathbf{x}W = 0$$

$W = 618$ N, $W_B = 180$ N

For (a), $B_y = 496$ N. Solving the equations for this case yields $\mathbf{x} = 1.511$ m

For (b), $B_y = 523$ N. Solving the equations for this case yields $\mathbf{x} = 1.611$ m

Problem 7.137 If a string is tied to the slender bar at A and the bar is allowed to hang freely, what will be the angle between AB and the vertical?



Solution: When the bar hangs freely, the action line of the weight will pass through the mass center. With a homogenous, slender bar, the mass center corresponds to the centroid of the lines making up the bar. Choose the origin at A, with the x axis parallel to the lower bar. Divide the bar into three segments plus the composite: (1) The segment from A to the semi circle, (2) the segment AB, and (3) the semicircle.

(1) $L_1 = 8$ cm, $\mathbf{x}_1 = 4$ cm, $\mathbf{y}_1 = 0$.

(2) $L_2 = \sqrt{8^2 + 8^2} = 11.314$ cm, $\mathbf{x}_2 = 4$, $\mathbf{y}_2 = 4$

(3) $L_3 = 4\pi = 12.566$ cm, $\mathbf{x}_3 = 8 + \dfrac{2(4)}{\pi} = 10.546$ cm, $\mathbf{y}_3 = 4$.

The composite length

$$L = \sum_1^3 L_i = 31.88 \text{ cm}.$$

The composite centroid:

$$\mathbf{x} = \frac{L_1\mathbf{x}_1 + L_2\mathbf{x}_2 + L_3\mathbf{x}_3}{L} = 6.58 \text{ cm},$$

$$\mathbf{y} = \frac{L_1\mathbf{y}_1 + L_2\mathbf{y}_2 + L_3\mathbf{y}_3}{L} = 2.996 \text{ cm}$$

The angle from the point A to the centroid relative to the lower bar is

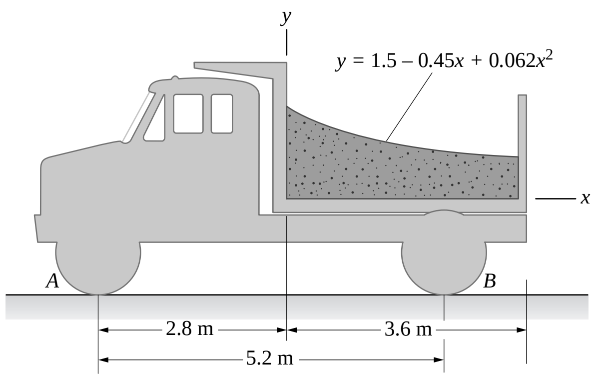
$$\alpha = \tan^{-1}\left(\frac{\mathbf{y}}{\mathbf{x}}\right) = 24.48^\circ.$$

The angle between AB and the lower bar is 45°, hence the angle between the line from A to the centroid and AB is

$$\beta = 45 - \alpha = 20.52^\circ$$

Since the line from A to the centroid will be vertical, this is the angle between AB and the vertical.

Problem 7.138 When the truck is unloaded, the total reactions at the front and rear wheels are $A = 54$ kN and $B = 36$ kN. The density of the load of gravel is $\rho = 1600$ kg/m^3. The dimension of the load in the z direction is 3 m, and its surface profile, given by the function shown, does not depend on z. What are the total reactions at the front and rear wheels of the loaded truck?



Solution: First, find the location of the center of mass of the unloaded truck (and its mass). Then find the center of mass and mass of the load. Combine to find the wheel loads on the loaded truck.
Unloaded Truck

$\sum F_x$: no forces

$\sum F_y$: $54000 + 36000 - m_T g = 0 (N)$

$\sum M_A$: $-\mathbf{x}_T m_T g + 5.2(36) = 0$

Solving $\mathbf{x}_T = 2.08$ m, $m_T = 9174$ kg

Next, find $\mathbf{x}_L$ and m_L (for the load)

$$\mathbf{x}_L = \frac{\int_{m_L} x\,dm}{\int_{m_L} dm} = \frac{\text{Num}}{m_L}$$

where $m_L = \int_{m_L} dm$, Num $= \int_{m_L} x\,dm$

$$m_L = \int_0^{3.6} 3\rho y\,dx = 3\rho \int_0^{3.6} (1.5 - 0.45x + 0.062x^2)\,dx$$

$$m_L = 3\rho \left[1.5x - 0.45\left(\frac{x^2}{2}\right) + 0.062\left(\frac{x^3}{3}\right)\right]_0^{3.6}$$

$m_L = 16551$ kg

$$\text{Num} = 3\rho \int_0^{3.6} (1.5x - 0.45x^2 + 0.062x^3)\,dx$$

$$\text{Num} = 3\rho \left[1.5\left(\frac{x^2}{2}\right) - 0.45\left(\frac{x^3}{3}\right) + 0.062\left(\frac{x^4}{4}\right)\right]_0^{3.6}$$

Num $= 25560$ kg · m

$$\mathbf{x}_L = \frac{\text{Num}}{m_L} = 1.544 \text{ m}$$

measured from the front of the load

The horizontal distance from A to the center of mass of the load is $\mathbf{d}_L = \mathbf{x}_L + 2.8 \text{ m} = 4.344$ m

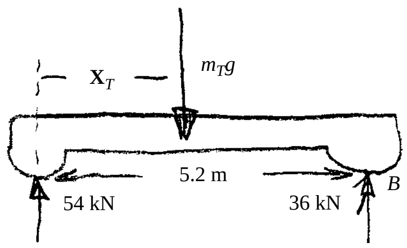
Now we can find the wheel loads on the loaded truck

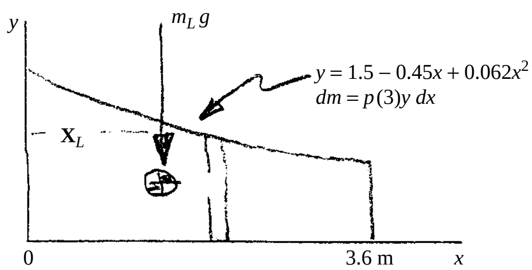
$\sum F_x$: no forces

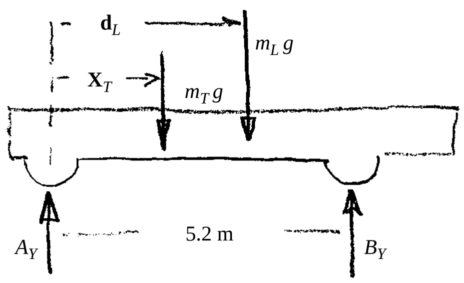
$\sum F_y$: $A_y + B_y - m_T g - m_L g = 0$

$\sum M_A$: $5.2B_y - \mathbf{x}_T m_T g - \mathbf{d}_L m_L g = 0$

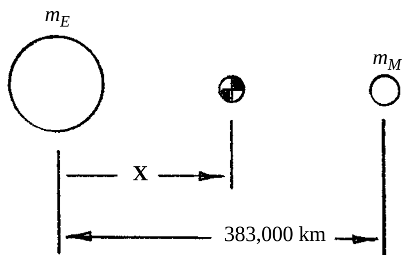
Solving $A_y = 80.7$ kN, $B_y = 171.6$ kN







Problem 7.139 The mass of the moon is 0.0123 times the mass of the earth. If the moon's center of mass is 383,000 km from the center of mass of the earth, what is the distance from the center of mass of the earth to the center of mass of the earth–moon system?



Solution:

$$\mathbf{x}(m_E + m_M) = 383{,}000\ m_M)$$

so $$\mathbf{x} = \frac{m_M}{m_E + m_M}(383{,}000)$$

$$= \frac{m_M/m_E}{1 + m_M/m_E}(383{,}000)$$

$$= \frac{0.0123}{1 + 0.0123}(383{,}000)$$

$$= 4650\ \text{km}.$$

(The earth's radius is 6370 km, so the center of mass of the earth-moon system is within the earth.)